

AN INVESTIGATION OF THE FACTORS WHICH INFLUENCE
THE FRICTIONAL PROPERTIES OF TEXTILE FIBERS

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SUMMARY

An investigation of the effects that certain parameters have upon the frictional properties of textile fibers has been conducted. The apparatus employed features a design which is capable of detecting minute forces between single fibers, drawn one across the other, and recording these forces by utilizing an electrical servo-system.

A systematic investigation of the apparatus and the inherent properties associated with it, indicated that the dynamics of the apparatus were a very important factor in the fiber friction investigation. Slow motion movies and stereomicroscopic observation indicated that with the present apparatus, a lever arm of a relatively high moment of inertia (12000 gm./cm.^2) gave more reliable frictional measurements than lower inertia arms. Because of the more consistent state of fiber to fiber contact, the high inertia lever arm maintained a more constant normal force. The rate of motion between the moving and static fibers as well as the dynamics of the response time of the frictional force detecting device were also concluded to be important features. The aforementioned factors are affected by the apparatus design, and all can affect the measured frictional forces between single fibers.

An investigation was made of the effect that varying the normal force had upon the coefficient of friction. It was found that in general the coefficient of friction (μ) decreased with an increasing normal force (N). It appeared, however, that μ was becoming asymptotic to a minimum value as N was increased to a point restricted by the tensile strength of the fiber. Cotton exhibited values for the kinetic coefficient

of friction over the range 0.200 to 0.300 as the normal force increased to 40 milligrams. The nylon used in this experiment gave values from 0.300 to 0.400 at this same level.

A similar decrease in μ was noted when the tension of the fibers in their holders was increased. With an increase in tension from 125 to 1150 milligrams, the kinetic coefficient of friction of cotton decreased from 0.356 to 0.236. Indications were that the rate of decrease diminished at successively larger loads, implying a similar asymptotic approach to a limiting value of μ .

Temperature cycling of cotton fibers in the range 70° to 220°C. to simulate gin drying of seed cotton resulted in small increases in the coefficients of friction. The net change was from 0.285 to 0.327 for kinetic friction and from 0.590 to 0.629 for the static value.

Different types of fibers exhibited data plots of different character specific to a given fiber and it appeared that the ratio μ_s/μ_k reflected an indication of the fiber's surface condition and/or shape. Convolutated cotton (with a relatively high static coefficient) gave μ_s/μ_k values in the range from 1.8 to 2.1. Rayon fibers gave a fairly consistent value at 1.8 and nylon dropped to about 1.6. It appeared that this ratio might be a constant for any given fiber of a cylindrical cross section. Hence, it is useful in determining μ_k values from the work of others in the fiber friction field, where the μ_s values alone has been reported.

CHAPTER I

INTRODUCTION

Statement of the Problem

The frictional properties of textile materials have been of interest to textile technologists for many years. As early as 1790, Monge¹ drew attention to the fact that wool fibers exhibited a differential frictional effect (D.F.E.) according to the direction of the scales due to the mechanical interlocking of the scales along the surface of the wool fibers. This theory, known as the ratchet theory, states that the scales can slide easily past each other in the root to tip direction but when reversed they interlock similar to the projections of a ratchet.

Friction is the principal property which holds fibers together in a spun yarn and subsequently binds these yarns together when interlaced into a fabric. Here, high friction is a definite advantage, because it makes possible the greater utilization of the inherent fiber properties into a desirable result -- a stronger yarn or a fabric with high dimensional stability. In some cases friction tends to be a disadvantage. The best known example is that of a yarn passing over a guide. High friction will cause excessive wear on guides, overstraining the yarn which in turn causes permanent damage which may result in unevenness or excessive breakage.

W. L. Balls in 1928 emphasized the importance of friction in textile processes when he stated: "up to the front mule roller, cotton must be slippery; afterwards it must be sticky."²

In more recent times the interest in the frictional properties of textile materials has increased considerably. With the advent of high speed equipment and man-made fibers, there has arisen a critical need for an account of textile friction and its causes and effects as related to textile processing and final properties of the finished product.

In textile processes such as opening, carding, drawing, and spinning, the role of friction is vitally important. Such attributes as breaking strength, elasticity, handle, and abrasion resistance in the final product are influenced by the frictional characteristics of the fibers or yarns from which it is composed.

Before the frictional behavior of fiber assemblies can be thoroughly understood it is necessary to understand the frictional behavior between single fibers. An investigation of the various parameters which influence the friction between single fibers will prove invaluable in determining their behavior during mechanical processing and the influence they exert as a unit of an assembly such as a spun yarn.

An apparatus developed in part by T. E. McBride³ provided an instrument of sufficient potential sensitivity to give valuable measurements. However, several modifications were required on this basic apparatus in order to achieve the measurements desired. Certain variables in fiber mounting also existed which needed to be improved.

Specifically, the purpose of this research was to improve the basic friction measurement apparatus and to investigate parameters of fiber mounting and instrument operation which were critical to valid frictional measurements. It was planned to examine the frictional behavior of cotton and of several other fibers for studies that might reveal effects of shape as well as fiber material or treatment. Among the factors to be

studied were the effect of tension in each fiber, the effect of varying the normal force between the fibers, and the effect of temperature cycling on cotton fibers in a manner to simulate dryers used for seed cotton in gins.

Historical Development of Friction Theory

When one body slides across another there is a force exerted tangential to the surface resisting the motion. This force is called friction. The two classical laws of friction were initially discovered by Leonardo da Vinci⁴ and may be expressed in the following manner:

1. The frictional force F is proportional to the normal load N such that the coefficient of friction $\mu = F/N$ remains constant for a given pair of bodies.
2. The frictional force is independent of the area of contact between the two bodies.

The French scientist Amontons⁵ re-discovered these laws in 1699. Coulomb⁶ verified them in 1788. Coulomb also pointed out that the force required to initiate sliding (static friction) is greater than the force required to maintain sliding (kinetic friction).

Coulomb also considered the possibility that friction arises essentially from the asperities present on all surfaces, the frictional work being expended in lifting the asperities of one surface over those of the other. During the nineteenth century, investigators generally confirmed Coulomb's experimental results and accepted his roughness theory of friction.

During the past twenty-five years several theories of friction have been proposed or developed but in effect they all fall either into the category of Coulomb's surface roughness theory or the surface interaction theory which involves adhesion or welding as noted by Bowden and

Tabor.⁷ The adhesion theory was originally developed for metals and considers adhesion between solids to occur as a macroscopic phenomenon analogous to "cold-welding." This mechanism to date has proven to be the most valuable of the two theories in explaining observed phenomena of friction in non-metallic solids.

The classical laws of friction do not apply to textile fibers as strictly as they do to metallic substances. Many theories have been offered by textile scientists for this deviation, but in general the frictional properties of the fibers are ascribed to a modified adhesion model. The major factor affecting the friction between polymers is the area of contact. This depends on the geometry of the surface and on the scale of surface roughness as well as on the load and in some cases a time factor. Natural textile fibers tend to have an undefined area of contact since in most cases their linear density and shape varies. For these convolutions, crimp, and inherent variability make the task of determining the area of contact difficult if not impossible. For synthetic fibers more uniform shapes may be obtained but other features intrinsic to polymers exert some influence. The fibers are visco-elastic and the area of contact depends on the time of loading and on the speed of sliding as well as upon shape.

Measurement of Friction Between Single Fibers

A search of the literature reveals that the body of published information on fiber friction is quite large, but the practical data available on any one phase of the subject is quite limited.

Howell, Mieszkis, and Tabor⁸ have recently published a book which deals exclusively with textile friction. This volume contains a summary

of the many methods used in the measurement of friction in textile materials as well as typical results reported by various investigators.

To further add to this body of information, two articles have appeared in trade journals which give reviews of the literature on fiber friction.^{9,10}

T. E. McBride¹¹ made an exhaustive literature search on the subject and reports over one hundred references on the subject of fiber friction.

An examination of the literature reveals generally that fiber friction can be measured in four different ways:

- (1) Friction between two single fibers;
- (2) Friction between one single fiber and one or more fiber assemblies;
- (3) Friction between two fiber assemblies; and
- (4) Friction between one single fiber and an unrelated surface.

When considering the case of friction between two single fibers there exists several methods to accomplish this. One utilizes the fiber twist method in which two fibers are twisted and the force necessary to induce slippage is taken to be an indication of the coefficient of friction.

The other and perhaps the most common method of measuring friction between single fibers involves the method utilizing friction with a single point of contact. The system used in this investigation employs this method. The following discussion is a review of the reported literature of this limited field.

A system very similar in principle to the one used in this research has been reported by Guthrie and Oliver.¹² The essential features of this

apparatus are shown in Figure 1.

The principle depends on one fiber being attached to a lever arm suspended by a torsion wire with another fiber moving across it at a given angle and fixed speed. The resulting frictional force between the two fibers causes the torsion wire to be twisted. The fixed fiber sticks to the traversing fiber until the force developed increases to a point that the fiber surface adhesion can no longer resist the counter-torque of the wire and slippage occurs. A similar behavior has been discussed by Bowden and Tabor.¹³ It is stated that at the moment of slippage $F = AS$ where A is the contact area and S is the shear strength of the polymer. The fibers will continue to slip until the countertorque is near zero and the sticking phase begins again. The process is repeated over and over giving a series of "stick-slips" that characterize the friction between the two fibers. Deflections of the upper (fixed) fiber were recorded by Guthrie and Oliver by means of a beam of light reflected from the mirror, attached to the wire, and focused onto a moving photographic film.

The fibers were mounted on detachable fiber holders which enabled easy mounting and tensioning. Guthrie and Oliver are the only known investigators who studied the effect of tension in the fibers using this particular type of apparatus. Their results on 3 denier viscose rayon at a normal force of 125 milligrams are shown in Figure 2.

A very similar method of measurement has been reported by Mercer and Makinson.¹⁴ This instrument is shown in Figure 3. Fiber K is mounted under slight tension, to a bow P which is attached to the end of a piece of clock-spring G, with a mirror F fixed to it. Fiber E is similarly mounted on the bow D which is fixed to one arm of the

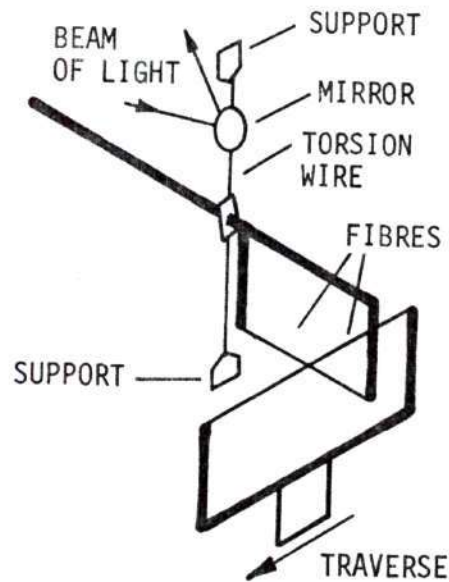


Figure 1. Essential Features of Guthrie and Oliver's Fiber Friction Apparatus.

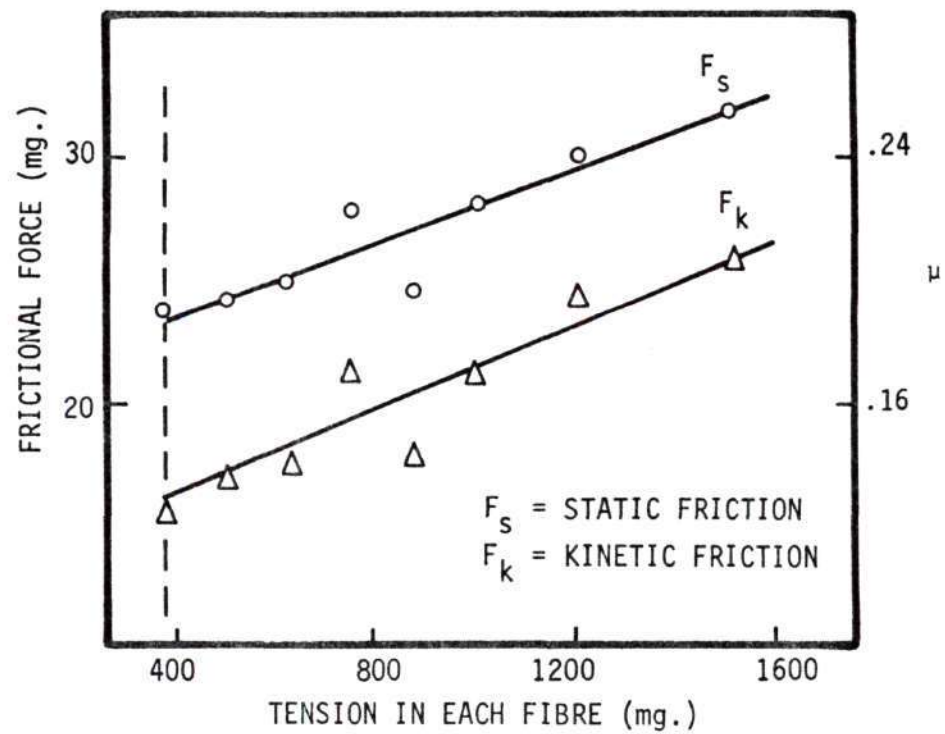


Figure 2. Results of Varying Tension on Viscose Fibers as Found by Guthrie and Oliver.

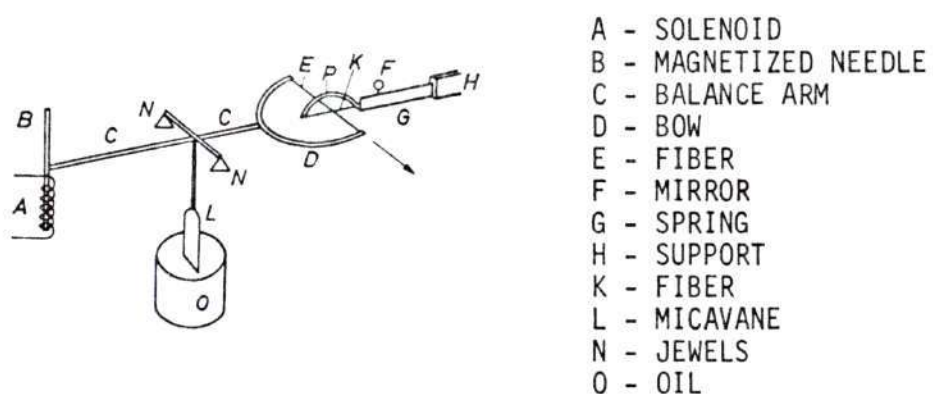


Figure 3. Mercer and Makinson's Fiber Friction Apparatus.

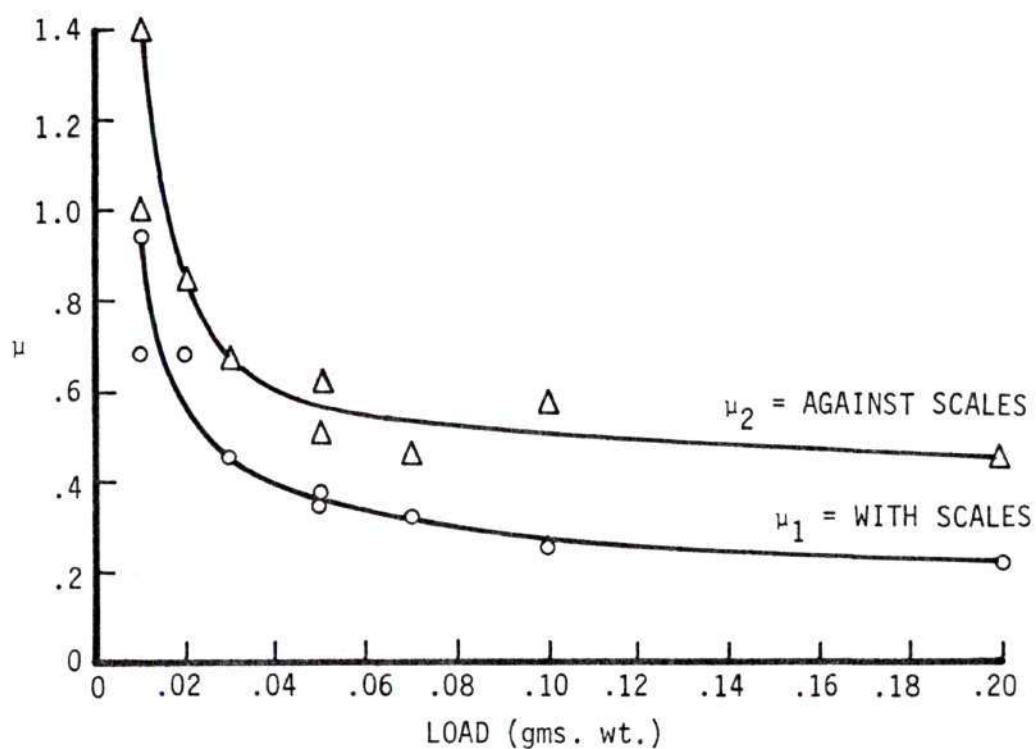


Figure 4. Variation of μ with Load on Wool Fibers According to Mercer and Makinson.

balance C supported on jewel bearings N. At the opposite end of the balance C a magnetized needle B is attached which is surrounded by solenoid A. A force can be applied by passing a current through the solenoid A. Vibrations are damped by the mica vane L dipping in the dashpot of oil O.

The fiber E is moved in the direction of the arrow by means of a hydraulic system with a velocity which can be varied between 0.01 cm./sec. and 0.1 cm./sec. The stick-slip traces are recorded on a film by a light source which reflects off the mirror F.

Mercer and Makinson were primarily interested in wool fibers but they did perform an investigation of the effects on the coefficient of friction caused by varying the normal load which was of interest to those engaged in the present investigation. These results are shown in Figure 4. It is also important to note that the normal force was applied electromagnetically, a method which appears superior to others.

A very similar apparatus to that of Mercer and Makinson was that adapted by Olofsson and Gralen for use on wool and viscose rayon fibers.¹⁵ This paper discussed the dependence of frictional forces on the area of contact, relative velocity, and the normal forces between the fibers.

Hood¹⁶ has investigated the frictional properties of several types of fibers using a fiber twist method. Even though this apparatus does not utilize a single point of contact technique, the results he obtained by varying the tension in each of the fibers were of interest. These results are shown in Figure 5. On the vertical axis of this figure the reciprocal of the number of turns per inch of twist inserted in the fibers is plotted. This parameter is directly proportional to the coef-

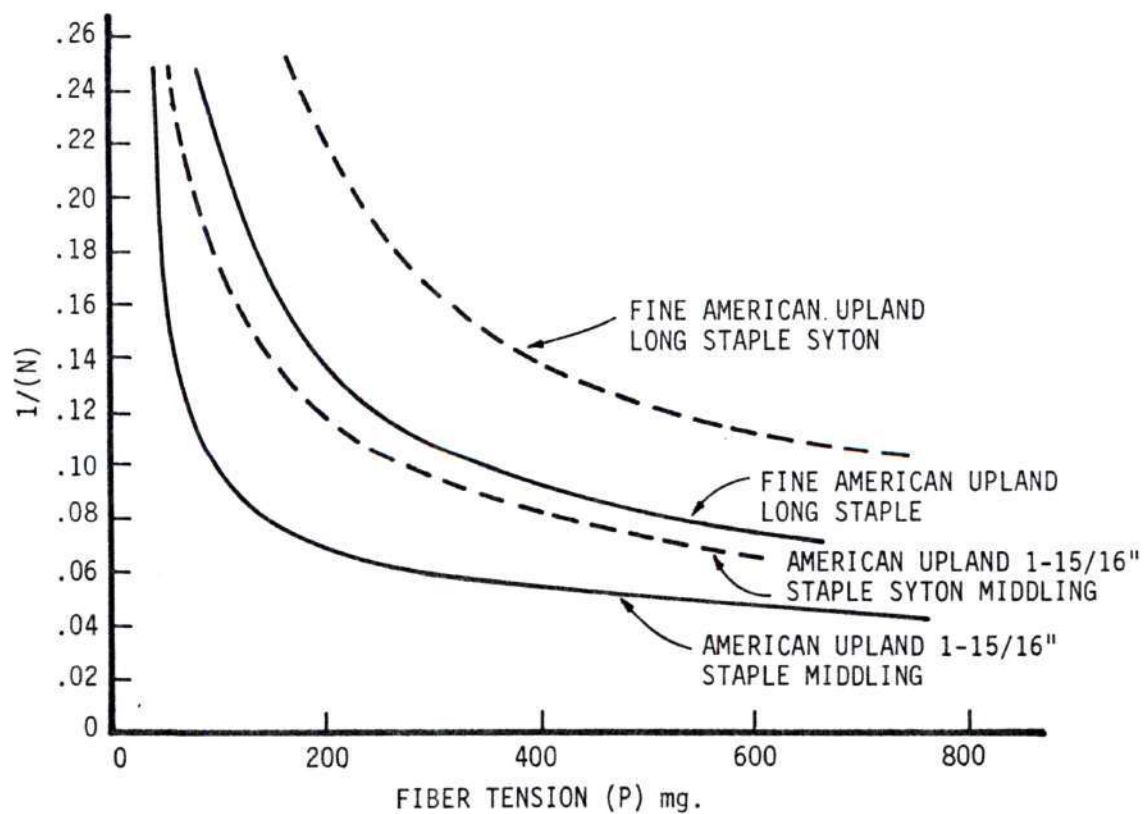


Figure 5. Results of Varying Tension on Cotton Fibers as Found by Hood.

ficient of friction. It will be noted that $1/N$ decreased with an increase in tension.

Howell has devised a rather unique method of measuring the friction between single fibers.¹⁷ This apparatus is shown in Figure 6. The unique feature of the instrument is that the normal force and the frictional force are calculated from a triangle of forces method. In the figure the horizontal fiber can be moved up to the vertical fiber by means of a screw operating the platform C. As the platform is moved forward, the normal force between the two fibers increases. The results of the effect of varying the normal force on the coefficient of friction are shown in Figure 7.

From an investigation of the literature it is evident that no single standardized method or procedure has been used for measuring fiber friction.

However, evidence has been presented by the authors cited, that the coefficient of kinetic friction between textile fibers lies in the range 0.14 to 0.40 for most cases and that the value depends upon both the tension in the fibers and the normal load. For tension changes in the range from 400 to 1600 milligrams, Guthrie and Oliver found that the coefficient of kinetic friction increased by a factor of approximately 1.7 for viscose rayon.¹⁸ Hood varying the tension in cotton fibers in a tension range from 30 to 800 milligrams showed a decrease in friction with an increasing tension.¹⁹ However, the significance of his results was that the greatest proportion of the change came at tensions below 200 milligrams.

Mercer and Makinson presented evidence that the coefficients of

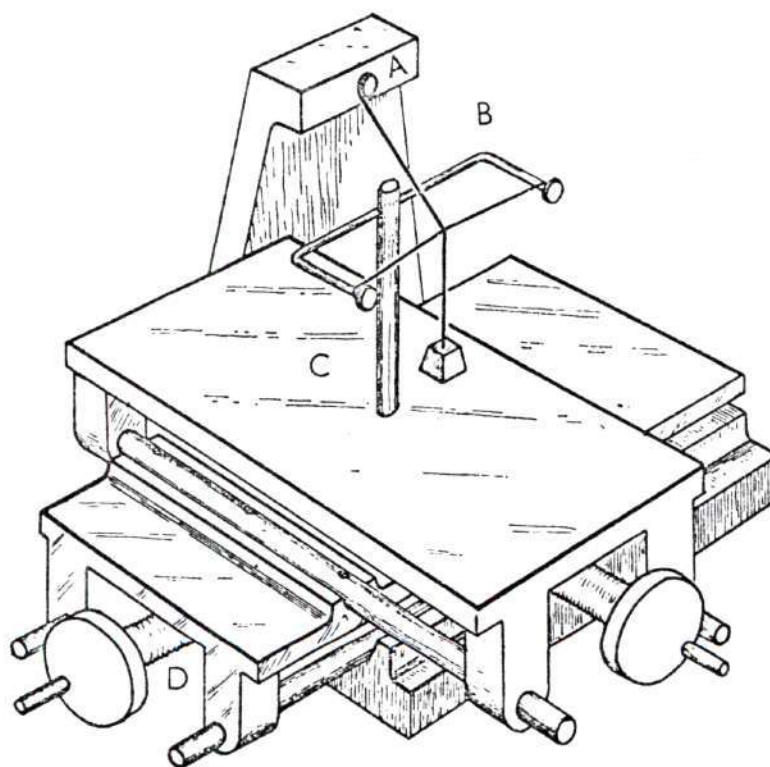


Figure 6. Essential Features of Howell's Instrument.

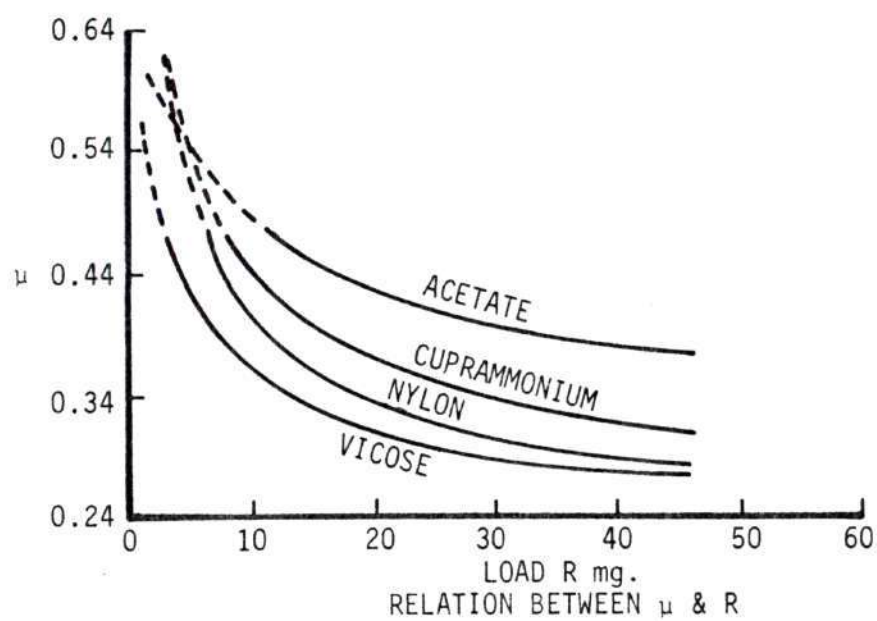


Figure 7. Relation Between μ and Load as Reported by Howell for Several Fibers.

static and kinetic friction for wool varied with normal force, high values being registered at loads below 20 mg. whereas at loads in the ranges 40 to 200 mg. the coefficients exhibited only small negative slopes.²⁰ It is noteworthy here also that the ratio μ_s/μ_k was in the range 1.5 to 2.0. A similar result at load ranges of 5 to 50 mg. was reported by Howell for acetate, cuprammonium, nylon and viscose.²¹ It is shown in Figure 7 that for loads above 30 milligrams the coefficient of static friction decreases very little for these fibers.

CHAPTER II

APPARATUS AND EXPERIMENTAL TECHNIQUE

Fiber Friction Apparatus

A review of the literature shows a number of instruments which have been used to measure the frictional properties of textile fibers. Several of these instruments were described in the preceding chapter. Each instrument was evaluated and found to have certain disadvantages with respect to accuracy and flexibility. Each, in some way, was found to have features that were of merit. It was the desire of those concerned with this investigation to employ an instrument which incorporated the desirable features of previously reported instruments, correct their major faults, and to feature some new innovations. The result was a highly sensitive apparatus capable of dealing with small fibers, measuring forces of a few milligrams accurately, and presenting data in an easily interpreted form.

T. E. McBride²² has previously discussed the basic design and construction of this apparatus in detail. The apparatus employs a torque principle and is similar to the one described by Guthrie and Oliver.²³ A diagram of the assembled, experimental instrument is shown in Figure 8.

Consideration of several methods of mounting a fiber and imparting a slow linear motion to it led to mounting a fiber on the end of a long rod which acted as a balance arm. The rod was pivoted in sapphire bearings about a point approximately ten inches from the mounted fiber. A counterbalance was used on the opposite end of the rod for adjusting the

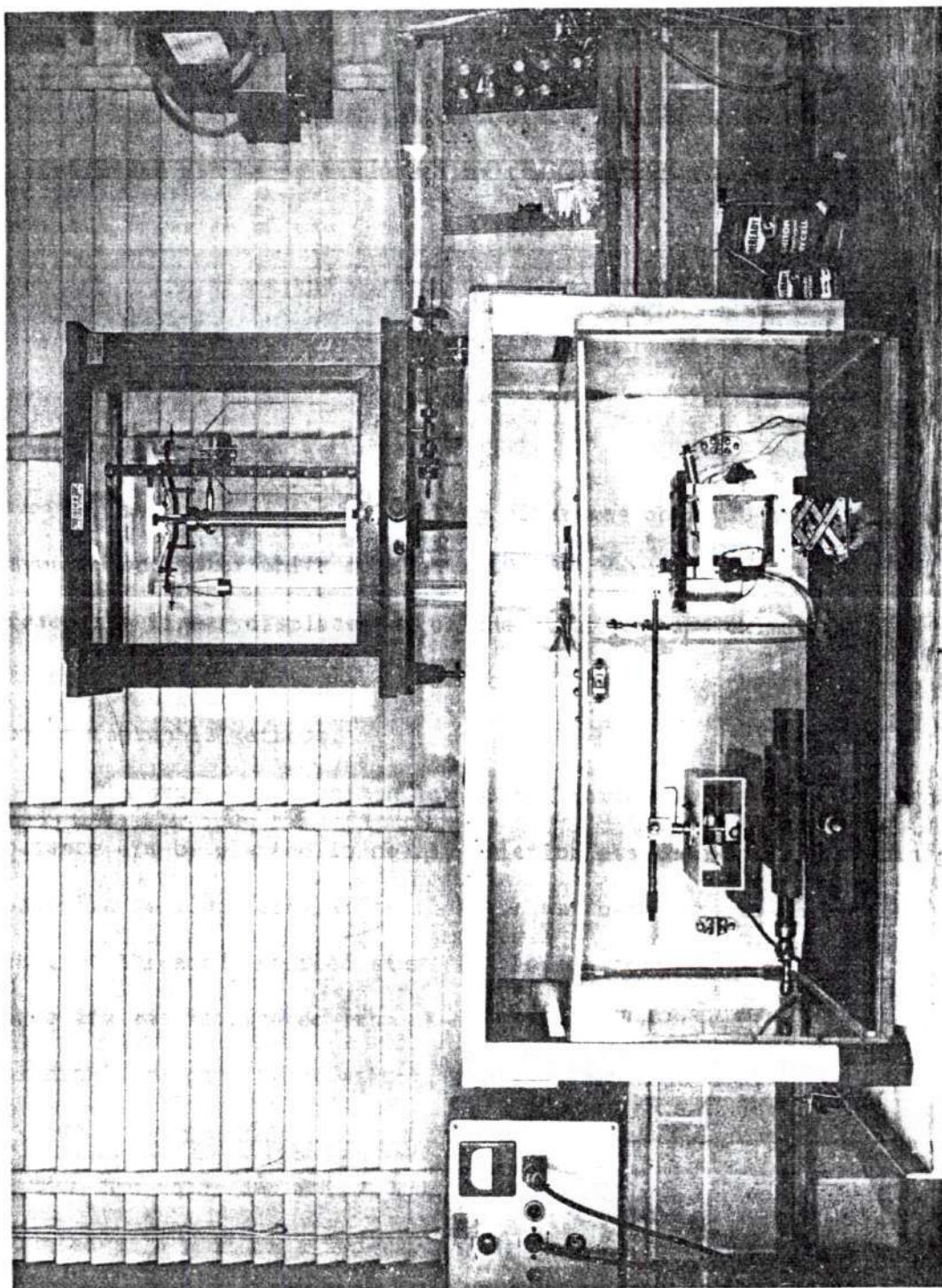


Figure 8. View of Frictional Measuring Apparatus and Chainomatic Balance for Measurement of Normal Force.

normal force. By rotating the arm and subsequently the fiber, an approximately linear motion was imparted to the fiber.

Use of the lengthy (39.3 cm.) arm was necessary to prevent any significant arc being traversed by the fiber in its short movement. The maximum traverse of the fiber was limited to less than three-fourths of an inch. Therefore the maximum deviation from a linear path was less than one thirty-second of an inch. Although the effect of this is not negligible, it is small compared with other problems associated with the measurement.

The speed of the traversing fiber was obtained by gearing down a synchronous motor which rotated at 0.0267 revolutions per minute. Therefore, the linear displacement of the fiber was approximately 0.11 mm./sec., a speed slow enough to make practical the study of the stick-slip effect by photographic methods.

In order to establish low normal forces it was essential that the balance arm be pivoted in nearly frictionless bearings. For this purpose, sapphire bearings similar to those in watches, were used and set in the ends of threaded, knurled studs in the arm support, Figure 9. The balance arm was fabricated from K-Monel tubing which exhibited properties of high strength, light weight, and was not influenced by stray magnetic fields.

The motor and drive, the balance arm, and a fiber holder were mounted as a unit on a two-way milling vise which made possible motion of the arm and fiber assembly both parallel and perpendicular to the axis of the balance arm. Therefore, the fiber could be placed at any desired position on a second fiber which was mounted on a galvanometer needle.

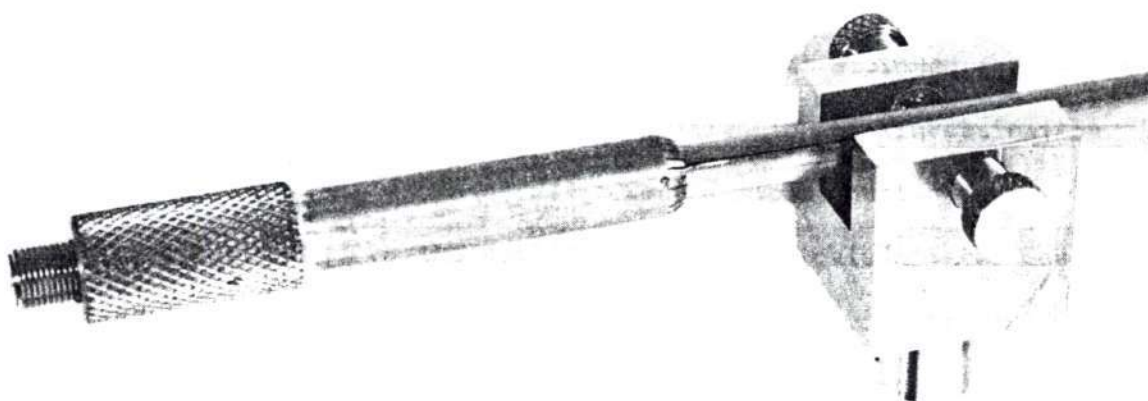


Figure 9. Balance Arm Support and Counterbalance Mechanism.

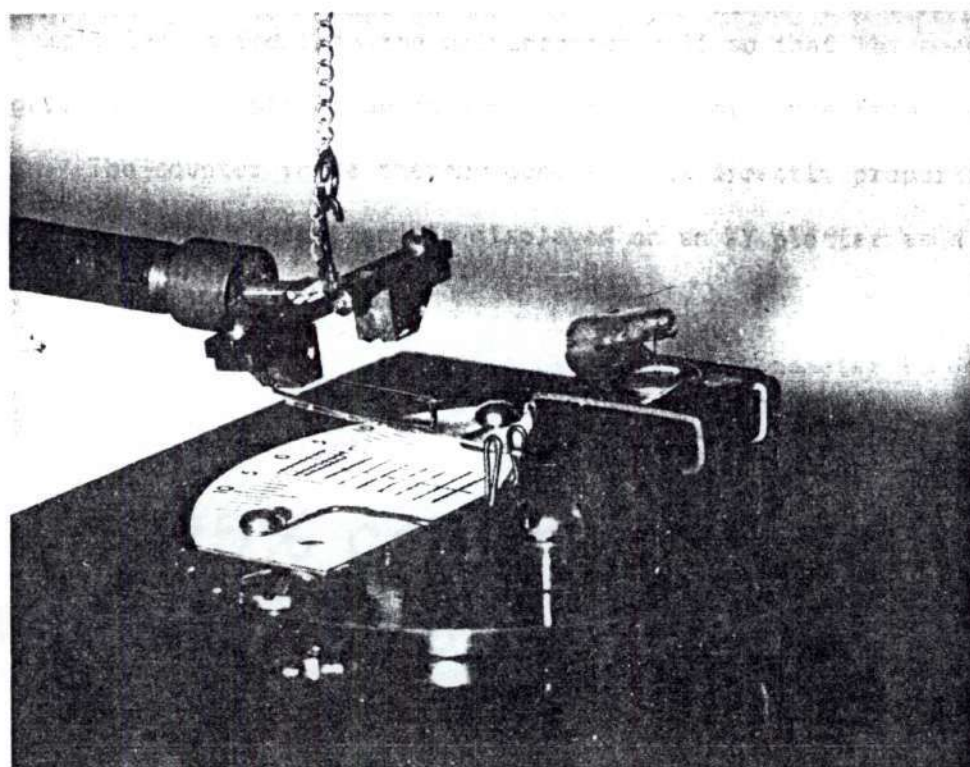


Figure 10. Close up of Frictional Arm Exhibiting Attachment of Chain to Balance.

The second fiber was mounted in a bow type holder which was fastened to the galvanometer needle by sealing wax.

An automatic, self balancing servo-system is employed for detecting and recording the minute forces encountered in this investigation. The traversing fiber is drawn across a second fiber mounted on the indicating needle attached to the coil of a D'Arsonval galvanometer movement. On displacement current is generated by the sensor, amplified and passed through the galvanometer coil to supply a balancing counterforce. The sensing mechanism consists of a system employing a beam of light reflected from a mirror on the suspension system of the galvanometer to a dual photo-diode which detects the slightest shift of the light beam. The signal from the photo-diode is fed into a high gain servo-amplifier. The output of the amplifier is fed into the galvanometer coil so that the needle is not significantly displaced by an applied frictional force from the null position. The counter force thereby generated is directly proportional to the galvanometer current and is displayed on an XY plotter as a function of the displacement.

The system was calibrated by rotating the galvanometer 90 degrees from the normal vertical position and hanging small known weights on the needle at the point where the fibers normally were in contact.

Fiber Mounting Technique

Inasmuch as several factors caused obvious deviations in the measured frictional forces between fibers, a consistent fiber mounting technique was required. A method was designed to attach a fiber in its holder so that its axis was parallel to that of the galvanometer needle.

A special holder was also constructed for mounting a second fiber on the end of the balance arm at any tension desired (see Figure 10).

For the fixed fiber, a piece of eight-thousandths phosphor bronze was made into a U-shaped holder. During mounting, the holder is placed in a special vise. One end of a fiber was affixed to a desired weight by sealing wax and the other end was glued to one side of the holder with Duco cement. When the cement had dried, the fiber was allowed to hang free with the weight attached and was promptly cemented to the other side of the holder. The holder was then fastened to the galvanometer needle with a small mass of sealing wax.

The holder for the fiber on the balance arm was removable and the fiber was affixed in a similar manner as before. This holder is fabricated from brass and stainless steel stock. Minimum dimensions were used to prevent increasing the total mass of the arm and thus prevented excessive friction in the bearings. The holder was designed to handle a range of lengths by incorporating adjustable screws which operated in a small slot. A close up view of both fiber holders is shown in Figure 10.

The pair of mounted fibers were not changed until three consecutive frictional measurements were completed. No significant change in data was noticed as a result of this procedure as opposed to changing both fibers for every experiment. However, fibers of corresponding lengths were used in all measurements in both fiber holders.

Testing Procedure

The initial step in testing the fibers was the determination of

the normal force by the chainomatic balance. Care was taken to insure that the lever arm was in the level operating position at the time of the measurement. A vernier height gauge was employed for this purpose. After the normal force was determined, a reference line was inscribed by the XY plotter on the data sheet while the fibers were not in contact. The galvanometer assembly was raised until contact between the fibers was made with the lever arm in the level position; a second reference line was then plotted. The purpose of the second reference line was to provide a check on the proper alignment of the fiber holder on the galvanometer needle. Ideally the two lines should coincide but misalignment caused a slight deviation.

With the fibers in operating position and when a visual check determined the absence of oscillation in the arm, the XY plotter was turned on almost simultaneously with the synchronous motor which started the arm. As the traverse took place the stick-slip traces were recorded for a length of almost 0.5 inch on the traversing fiber. This corresponded to approximately 6 to 10 inches on the data sheet using a chart pen speed of 0.1 inch per second and a pen sensitivity of 5 millivolts per inch for cotton fibers.

At the end of the first run the fibers were repositioned and the operation repeated until a total of three traces were obtained for each pair of fibers.

After the third trace the normal force was again determined and the arithmetic mean of the two weighings was taken as the normal force for the three measurements.

Interpretation of Data

After the frictional data for fibers had been obtained under various conditions, the data sheets from the XY plotter were analyzed. By using planimeter integration, the area under each curve was calculated. Dividing this quantity by the length of the base line, obtained by inscribing a line on the chart while the fiber was at rest on the needle, gave the average deflection of the needle. From calibration data of the system, this deflection was converted into the kinetic frictional force expressed in milligrams. The normal force between the fibers was measured before and after each sequence of tests and a numerical mean obtained. This quantity was also expressed in milligrams. Therefore, a coefficient of kinetic friction was obtained by using the expression:

$$\mu = \frac{\text{Friction Force (mg.)}}{\text{Normal Force (mg.)}}$$

A coefficient of static friction was also obtained by determining the average height of the ten maximum deflections of the trace. This figure was converted into milligrams of frictional force and the coefficient of static friction was computed in the same manner as before.

Two other items of interest were calculated from the stick-slip traces. One was the ratio of the coefficient of static friction to the coefficient of kinetic friction (μ_s/μ_k).

The second item was the number of peaks per unit length of the traveling fiber. All results were tabulated and subjected to statistical analysis. The general character of the curves was also of interest as will be discussed subsequently.

Apparatus Modifications

Before and during the experiment, several modifications were made on the basic apparatus. A high fluctuation in the normal force readings led to the replacement of the fulcrum pen and bearings. This arrangement consists of a hardened steel pen (No. 33 pocket watch staff) mounted in sapphire bearings. The pen and bearings were designed to be more easily removed and replaced for periodic maintenance.

A rapid method of obtaining the normal forces between the fibers was devised. A stand was constructed for a chainomatic balance, as shown in Figure 8. This places the balance directly over the fiber holders. A chain suspended from a counterweight, replacing the left pan of the balance, extends down precisely to the level of the balance point of the respective lever arm where it can be engaged with a hook provided on the arm. A close-up view of this arrangement is shown in Figure 10.

Previously the light source had been powered by two 1 1/2-volt dry cell batteries. After a period of time, these batteries tended to weaken slightly and cause small errors in the frictional force readings. An A.C. electrical system was devised to correct this fault. After experimentation with several models, a very satisfactory 3-volt, 500 milliamp inductive choke filter power supply was employed. This arrangement also permitted a higher chart sensitivity as well as better stability of the chart pin at all input voltage ranges.

CHAPTER III

EXPERIMENTAL WORK

General

The cotton fibers used in this program were Empire WR grown at Experiment, Georgia. To eliminate effects caused by mechanical harvesting, hand picked samples were used. They were subsequently examined under a microscope to eliminate broken or otherwise damaged specimens. All fibers were measured by hand and separated into groups of 1/4 inch increments. Unless otherwise stated all frictional measurements were made on 1 1/4-inch fibers. McBride found no significant difference between 3/4, 1 and 1 1/4-inch fibers at a 95% confidence level.²⁴

The nylon used in the experiment on the effect of normal force was bright 15 denier monofilament manufactured by DuPont.

The studies made and discussed more completely on the following pages are the effect that the dynamics of the apparatus, the normal force, the tension, and heat exposure have on the measured coefficients of friction. In addition some comments on the stick-slip phenomena are discussed in the light of observations made by using slow motion movie photography and a stereo-microscope.

Effect of Dynamics of the System

During the course of this experiment it was realized that certain inherent dynamics of the system caused the measurement apparatus to behave

in a particular manner and consequently affect the results obtained. An investigation was made of the effect that the characteristics of the balance arm had on the frictional forces developed between two fibers. In addition to the K-Monel arm, two other arms were constructed. One was made of 1/8 inch aluminum rod, 29 cm. long, suspended at a position 23.9 cm. from the fiber holder. The other lever was constructed of 3/16 inch aluminum rod and was 31.8 cm. long, suspended at a point 22.9 cm. from the fiber holder. The mass of the three arms were 74.2 grams, 39.6 grams and 23.4 grams respectively and their moments of inertia were calculated to be essentially 12,000, 3,000, and 1,150 gm./cm.² respectively. A series of frictional measurements were made on cotton fibers (Figure 11) and it was observed that the K-Monel arm with the largest moment of inertia, gave minimum bounce and thus more uniform contact between the fibers during the measurement. This action indicated a constant and dependable normal force condition.

Since the shape and mass of the K-Monel arm gave it the higher moment of inertia, and this appeared to be an important factor in maintaining fiber to fiber contact, this arm was used in the remainder of the program.

The Stick-Slip Process

The occurrence of stick-slip or intermittent motion in the surface friction of fibers has been observed in this study. The electrical servo-system used portrays this motion very well in the form of peaked traces on an XY plotter. The upward motion of the trace is observed as a slow steady rise in the frictional force during the static or sticking

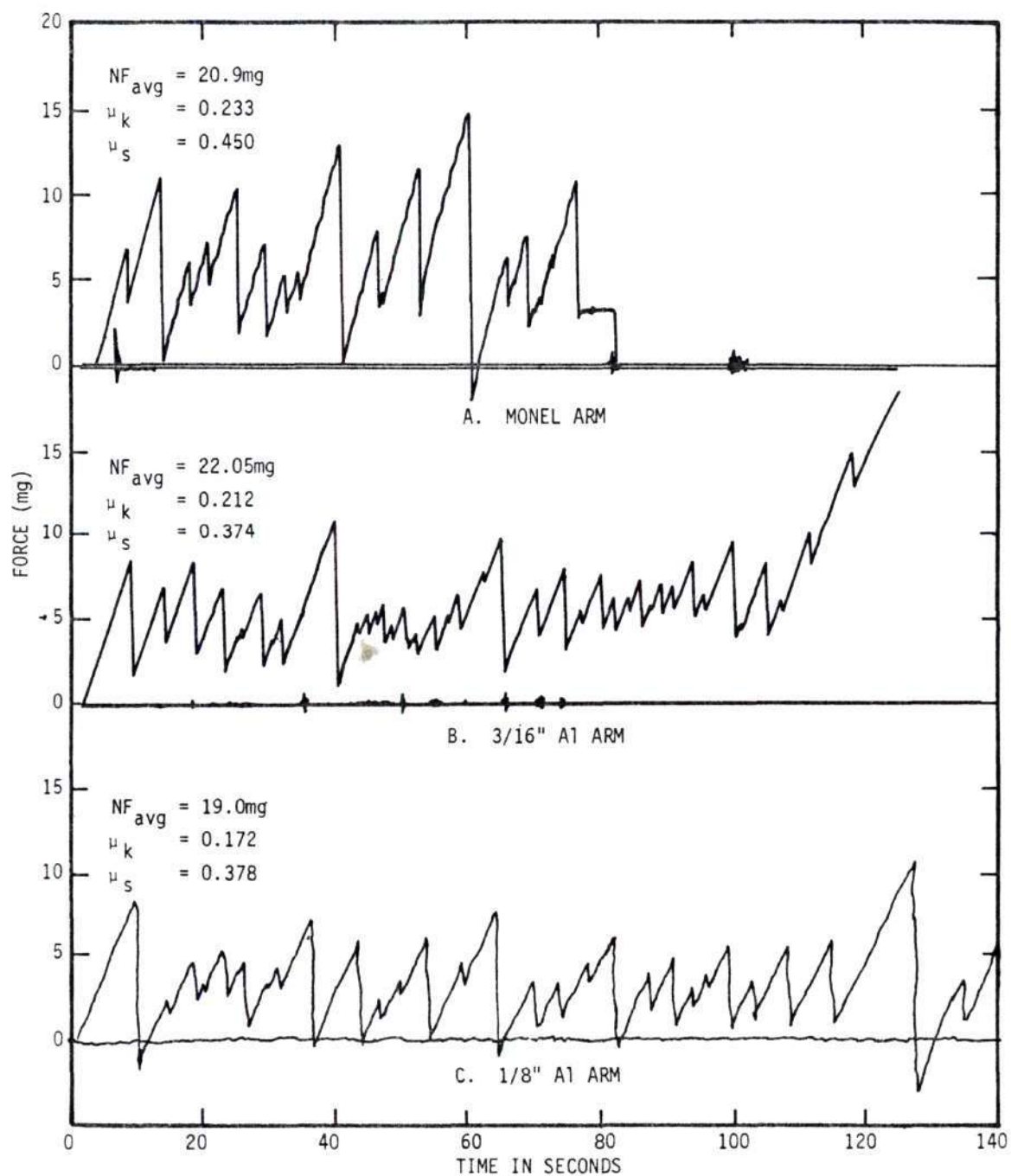


Figure 11. Comparison of Frictional Data Plots of 1 1/4-inch Empire WR Cotton Fibers Made with Three Different Arms at About 20 mg. Normal Force.

phase of the stick-slip process. Periodically, this force attains a value sufficiently high to overcome the static friction at the point of contact between the fibers. Then the fixed fiber slips and is returned toward the zero position by the restoring torque of the galvanometer system until the fibers stick again.

To better evaluate the instrument it was felt that a microscopic observation of the stick-slip motion would be beneficial. For this purpose, a Bausch and Lomb binocular stereomicroscope was procured. This microscope had adjustments for vertical and horizontal displacement so that the objective of the microscope could be placed extremely close (0.5 inch or less) to the fiber surfaces while the base and other working parts remained out of the way.

The wooden stand and plexiglass cabinet that encased the apparatus were arranged so as not to interfere with the microscopic observations. Due to the position of the galvanometer, the viewing area had to be approached from the rear side at an angle of about five degrees from the horizontal axis of the lever arm.

After experimentation the best viewing magnification was a combination of 10X eyepieces and a 3X objective for a total magnification of 30X. At this power an area of about 4 mm.² could be observed. The microscope was focused on the interaction point where the two fibers touched. Only fine adjustments had to be made on the depth of focus as the friction device was run.

Visual observations confirmed that the static portion of the curve was caused by the two fibers sticking together. However, the most interesting observations occurred during the kinetic portion of the cycle.

After the fibers slipped, the distance covered along the fiber axis until sticking reoccurred was quite large. It was suspected that over this region the fibers were not in complete contact. It was also apparent that the fibers twisted about their axes in such a manner as to enable the traversing fiber to follow a path of least resistance during its movement.

To enable better observance of the stick-slip motion, a series of slow motion movie films were taken of cotton and nylon fibers during typical frictional measurements. These films were made with a 16 mm. Bell and Howell Model 70 DH camera at 64 frames per second. This apparatus had a 4 inch focal length with an adapter for close-ups. The film was projected at 16 frames per second, a speed slow enough to detect discernible effects during the intermittent motion at an approximate total magnification of 15X. Action shots were taken of Empire WR cotton and 15 denier nylon at traverse speeds of 0.11 mm./sec. and 1.02 mm./sec. respectively, to determine the effect that traverse speeds might have on the stick-slip motion.

In general, it was noted that the higher traverse speed tended to introduce a higher periodic vertical oscillation of the arm and between the fibers due to the motion of the lever arm. This performance indicated a less consistent period of fiber contact as well as normal force fluctuations for the higher traversing velocity. The effect was especially noted for cotton fibers. The effect was less noticeable for nylon fibers, possibly due to their more regular surface and relatively higher denier (15 compared to approximately 1.5).

The motion picture films also depicted the same kinetic and static behavior as observed with the stereomicroscope. However, the most significant observation was that when slippage occurred the galvanometer needle, and thus the fixed fiber holder, returned to the null position, but that the fixed fiber itself was flexed toward the direction of movement by the traversing fiber and never quite returned to the null position. Since the servo-system detected only the movement of the galvanometer needle, this fiber displacement had some effects on the results. The system, however, may be incapable of detecting such small changes at this stage of its development.

Effect of Normal Force

A series of experiments were conducted on Empire WR cotton and 15 denier monofilament nylon to determine the effect that varying the normal force had on the coefficient of friction. A range of normal forces from 6 to 40 milligrams were used. The fibers were mounted under 425 milligrams tension and tested in the manner outlined in Chapter II. These measurements were conducted only for single fiber pairs of each material. From the equation $\mu = F/N$, the average coefficients of static and kinetic friction at a particular normal force were obtained. A plot of the μ_k values are shown in Figure 12.

Since the curves in Figure 12 appeared to behave in an erratic manner below a normal force of 20 milligrams, an additional series of experiments was carried out to determine the value μ_k using the large inertia arm (K-Monel) for additional fibers over the load range 10 to 40 milligrams. A plot of these values is shown in Figure 13. It is

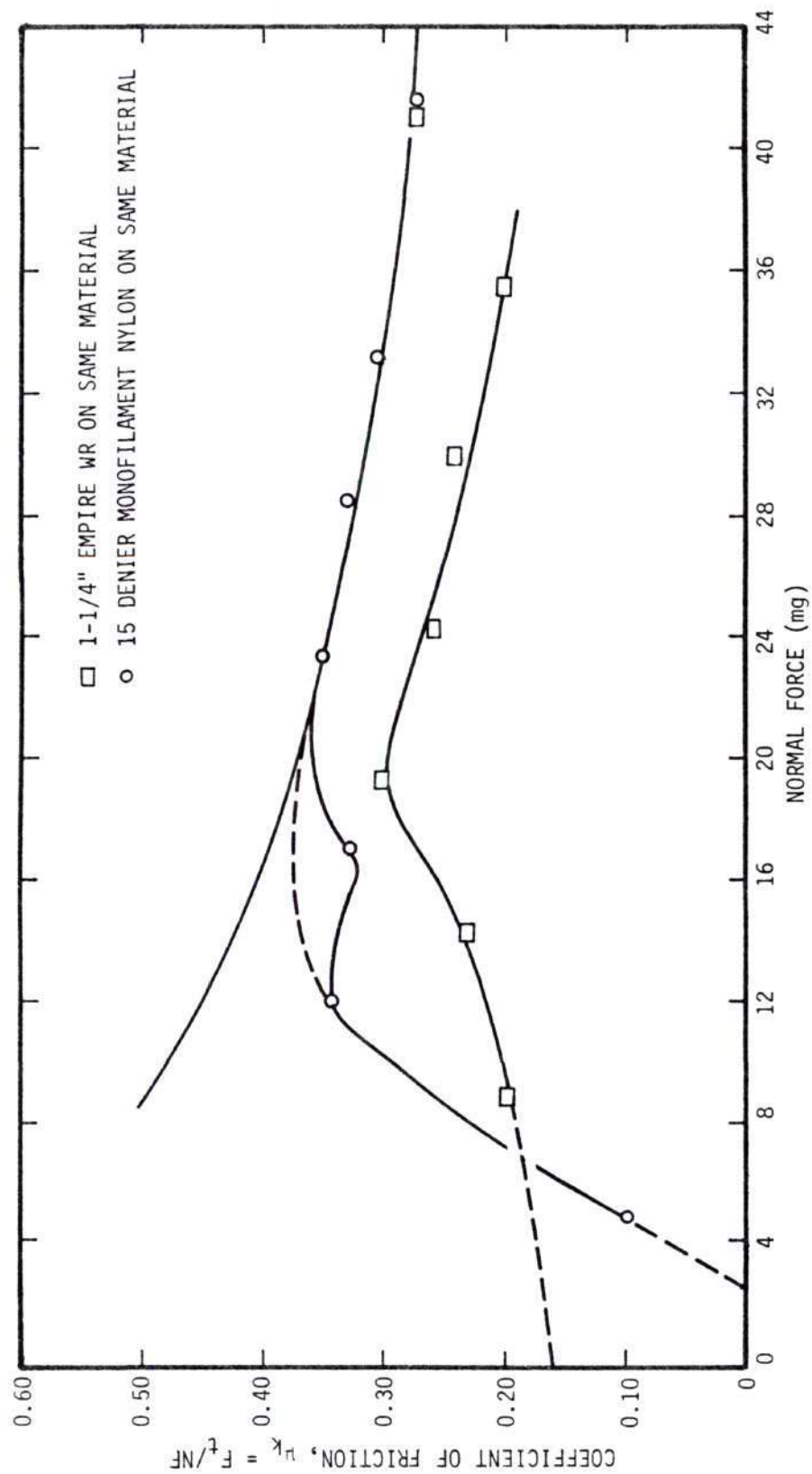


Figure 12. Plot of Coefficient of Friction Versus Normal Force Obtained for Single Fibers of 1 1/4-inch Empire WR Cotton and 15 Denier Nylon.

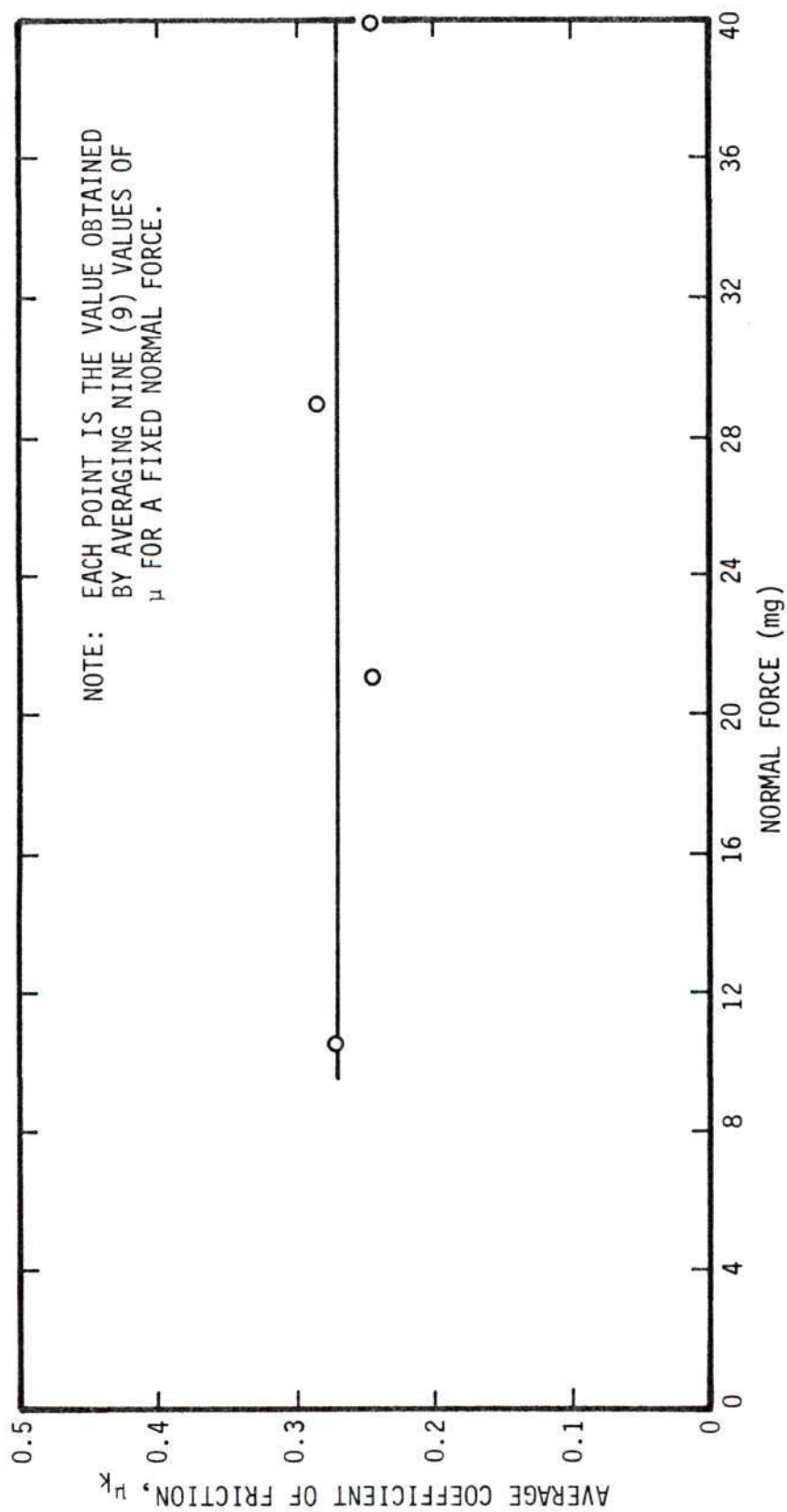


Figure 13. Plot of Average Coefficients of Friction Versus
Normal Force for 1 1/4-inch Empire WR Cotton Fibers.

evident from these data that there really does not exist a large variation with load in the range covered as determined with this apparatus assembly.

Effect of Tension

From the literature, and evidence accumulated during the experiments, it appeared that the initial tension or force under which the fibers were mounted in the two holders affected the results obtained. As a result direct measurements in statistically significant quantities were made of the effect of varying the tension on the coefficients of kinetic and static friction of Empire WR cotton.

A series of small weights were constructed to investigate this parameter. These weighed 125, 425, 825, and 1150 milligrams each. Fifteen fiber pairs were tested at each tension, and three frictional measurements were made for each pair at a normal force of 20 milligrams. It will be observed in Figure 14 that the value of μ_k ranged from 0.356 to 0.236 and μ_s from 0.647 to 0.483 over the tension range examined. This data and the statistical analysis are presented in Tables 2 through 8.

Photographs were taken of cotton fibers in a holder under each of the tensions involved in this experiment. These photographs are shown in Figure 15. The most apparent effect of applying a force, as observed from these photographs, is the disappearance of crimp. The behavior of the convolutions was not so readily discernible. This behaviour is discussed further in Chapter IV.

During this series of experiments an interesting effect on the

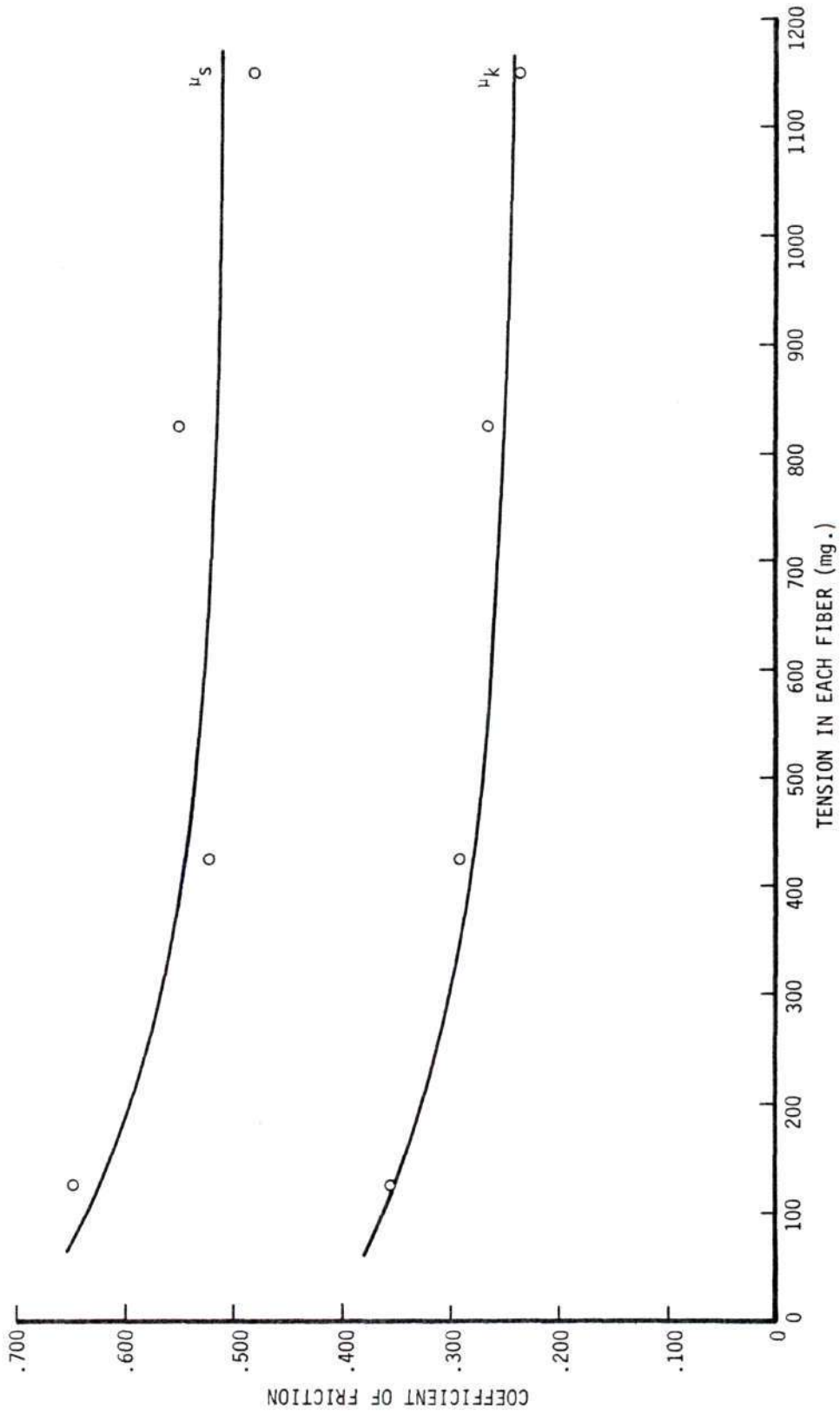
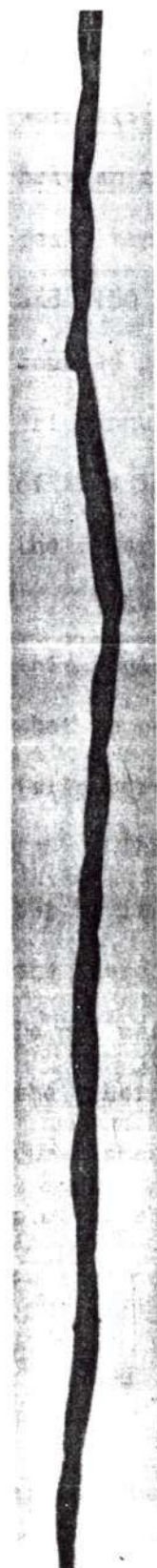


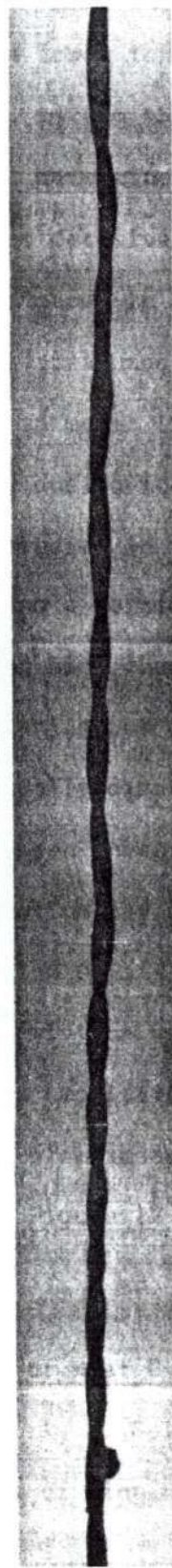
Figure 14. The Effect of Tension on the Coefficient of Friction of Empire WR Cotton.



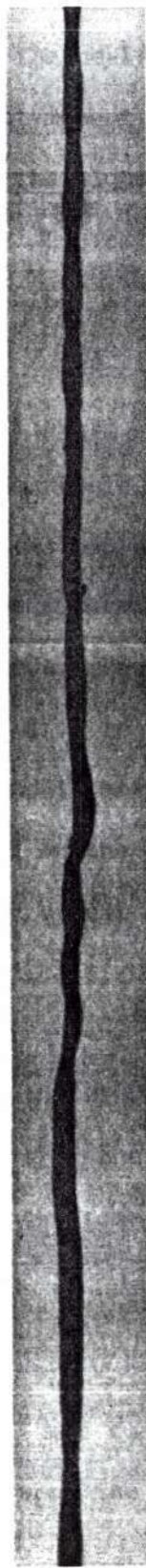
125 mg.



425 mg.



825 mg.



1150 mg.

Figure 15. Photomicrographs Empire WR Cotton Fibers at Successively Higher Tensions. (136x)

character of the obtained stick-slip curves was observed as a result of increasing the tension. At the lower tensions the peaks of the trace were slightly rounded in shape, making the exact moment of slippage between the fibers difficult to determine. At the higher tensions, the peaks tended to have a sharp and decisive slip. Typical traces at 125 and 1150 milligrams tension are shown in Figure 16. It can be seen that rounded peaks will increase the area under this curve, thus causing the frictional force measured by planimeter integration to increase. A part of this increase is undoubtedly due to the larger contact area between the fibers. However, further studies in this area would be desirable.

Lyons and Scheier,²⁵ using a torsion wire apparatus, have studied this region of slippage by using high chart speeds. They have reported what appeared to be a near-instantaneous linear drop in frictional force (slip portion of trace) was actually damped harmonic motion.

This phenomenon has not been investigated on the present apparatus, but it is suspected that such motion may occur. Visual evidence from the stereomicroscope investigation exists to indicate, however, that it is not pertinent to the actual friction value since the contact between the fibers is in question. This implication is that this portion of the curve should be ignored or very slow sweep speeds of the fiber and the plotter should be employed. In Figure 17 are shown some traces made at various pen speeds.

Effect of Heat Cycling

Concurrently with this work, A. Goldfarb²⁶ was concerned with the effect that conventional methods of ginning had upon the properties of

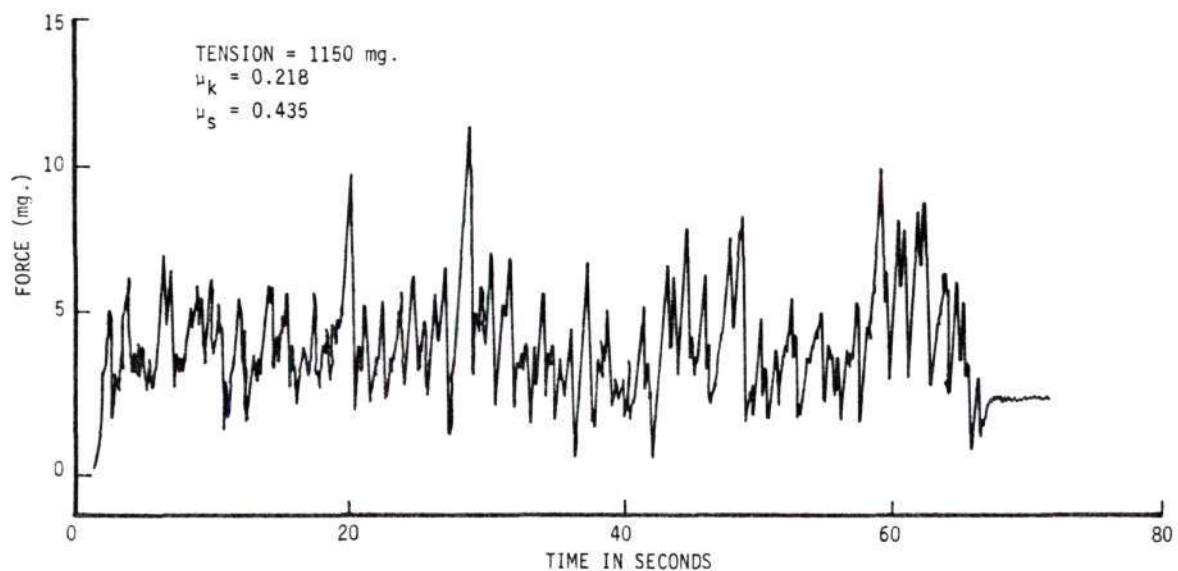
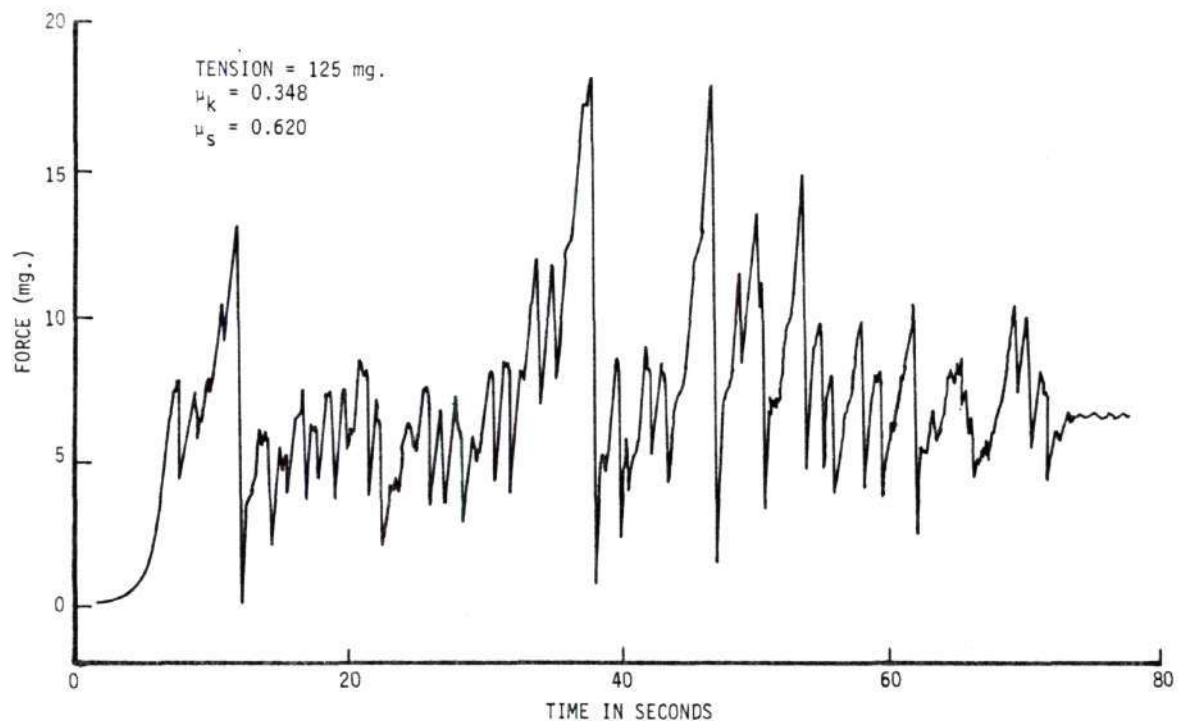


Figure 16. Frictional Traces of Empire WR at 125 and 1150 Milligrams Tension.

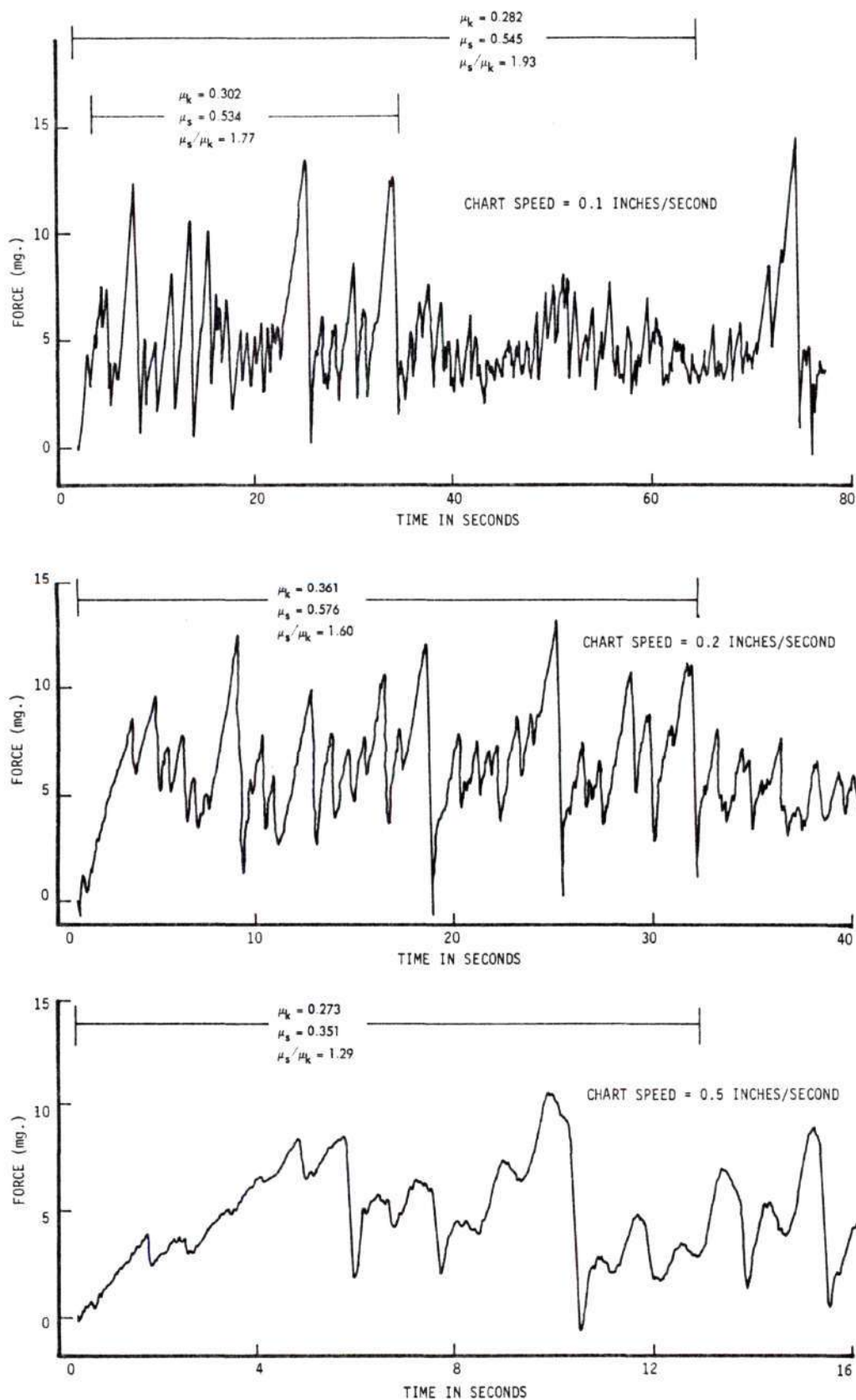


Figure 17. Typical Frictional Traces Made on Viscose at Various Pen Speeds.

cotton fibers. He was principally concerned with the physical properties of the cotton such as span length and fiber breakage as affected by the ginning process.

It was therefore of interest to determine the effect that gin drying temperature had upon the frictional properties of the cotton. A laboratory experiment was devised in which a group of hand ginned cotton fibers were placed into an oven and subjected to selected temperatures for a 15-minute interval.

The temperatures utilized in this experiment were 70°, 120°, 170°, and 220°C. This represented a range sufficient to include any conceivable temperature encountered in gin drying. The fibers were left in the oven for 15 minutes and then allowed to reach equilibrium by conditioning overnight in the laboratory.

The raw data and statistical analysis for this experiment are presented in Tables 9 through 15. The averages of the results are shown in Figure 18. The tension in each fiber was kept constant at 425 milligrams and the normal force at approximately 20 milligrams.

It will be noted that the coefficient of kinetic friction was increased as a result of heating from a value of 0.285 after heat cycling to 70°C to 0.327 after heating to 220°C. These values may be compared with the previous ones derived by Belser and Taylor²⁷ of 0.245 and the value 0.292 in the tension experiment reported previously for a normal force of 20 milligrams and a tension of 425 milligrams.

Miscellaneous Studies

Preliminary testing of fibers other than cotton have produced

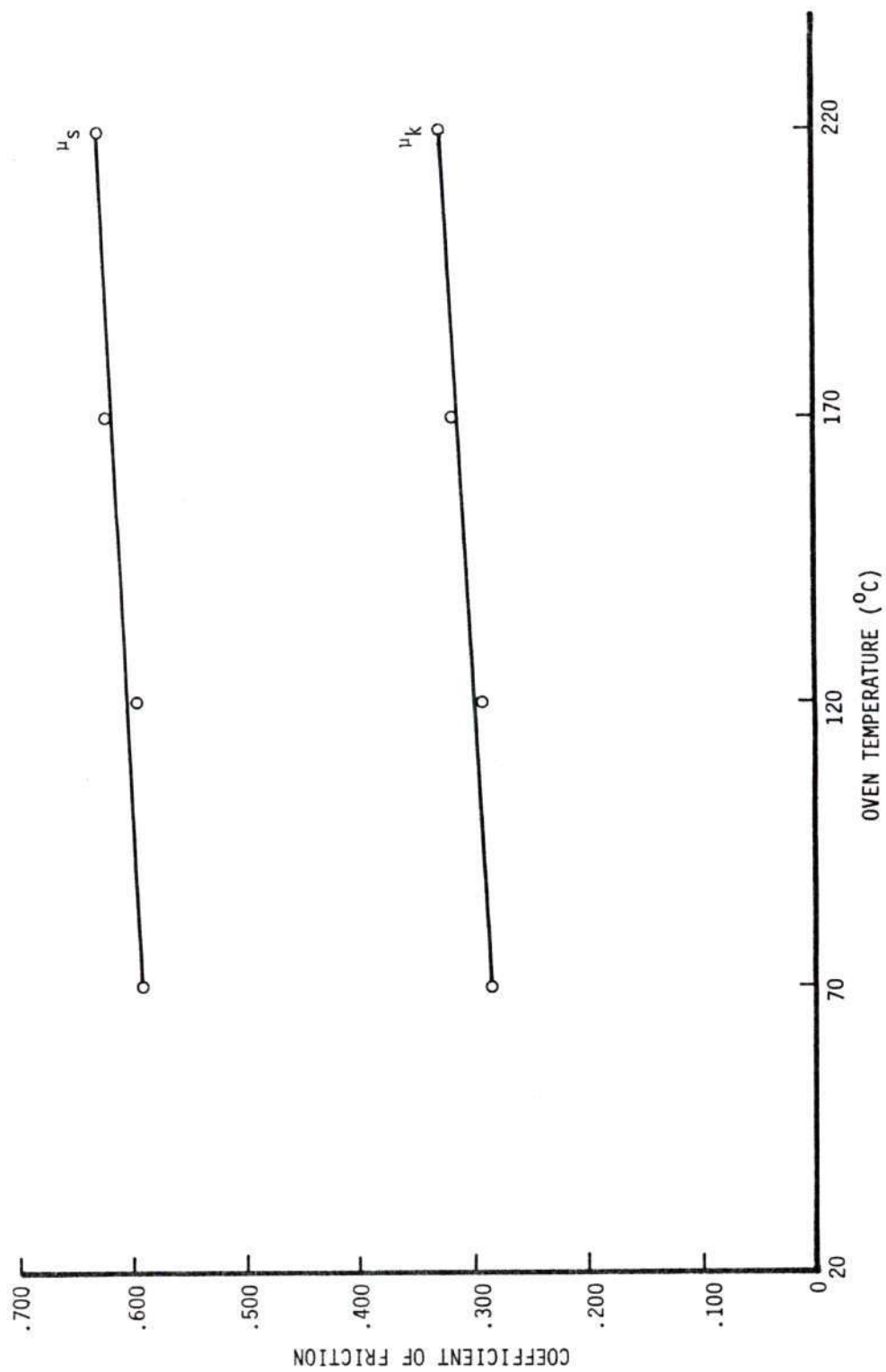


Figure 18. Effect of Heat on the Coefficient of Friction of Empire WR Cotton.

frictional coefficients which are comparable to those reported in the literature. Limited measurements performed on Courtauld's 1.5 denier regular rayon staple fiber have indicated a value for μ_k of 0.304 and of μ_s of 0.539 at 20 milligrams normal force. It was noted that these regenerated fibers exhibited a definite lack of periodic high irregularly spaced static peaks resulting in a fairly consistent μ_s/μ_k ratio of approximately 1.8. This was also noticeable for DuPont's 15 denier monofilament nylon which gave a value of 0.338 for μ_k and a value of 0.549 for μ_s . The μ_s/μ_k ratio for nylon was found to be approximately 1.6 when using a normal force of 20 milligrams. Typical plots of frictional data for cotton, rayon, and nylon are shown in Figure 19.

A novel experiment was conducted on Empire WR cotton fibers which had been coated with aluminum in a vacuum system. The purpose of this experiment was to evaluate the effectiveness of the friction apparatus in detecting surface abnormalities on textile fibers. This was considered to have future applications in the investigation of mechanical damage to fibers and the effects of delusterants and lubricants in the case of man-made fibers. The values of μ_k and μ_s for these fibers were 0.378 and 0.653 respectively with a μ_s/μ_k ratio of 1.75. These values may be compared with 0.292, 0.523 and 1.8 for the μ_s/μ_k ratio for untreated cotton fibers. Figure 20 shows a comparison between treated and untreated cotton fibers.

The values of the frictional coefficients for cotton, rayon, and nylon fibers obtained by the writer are somewhat higher than most of the values reported in the literature and summarized in Table 1.

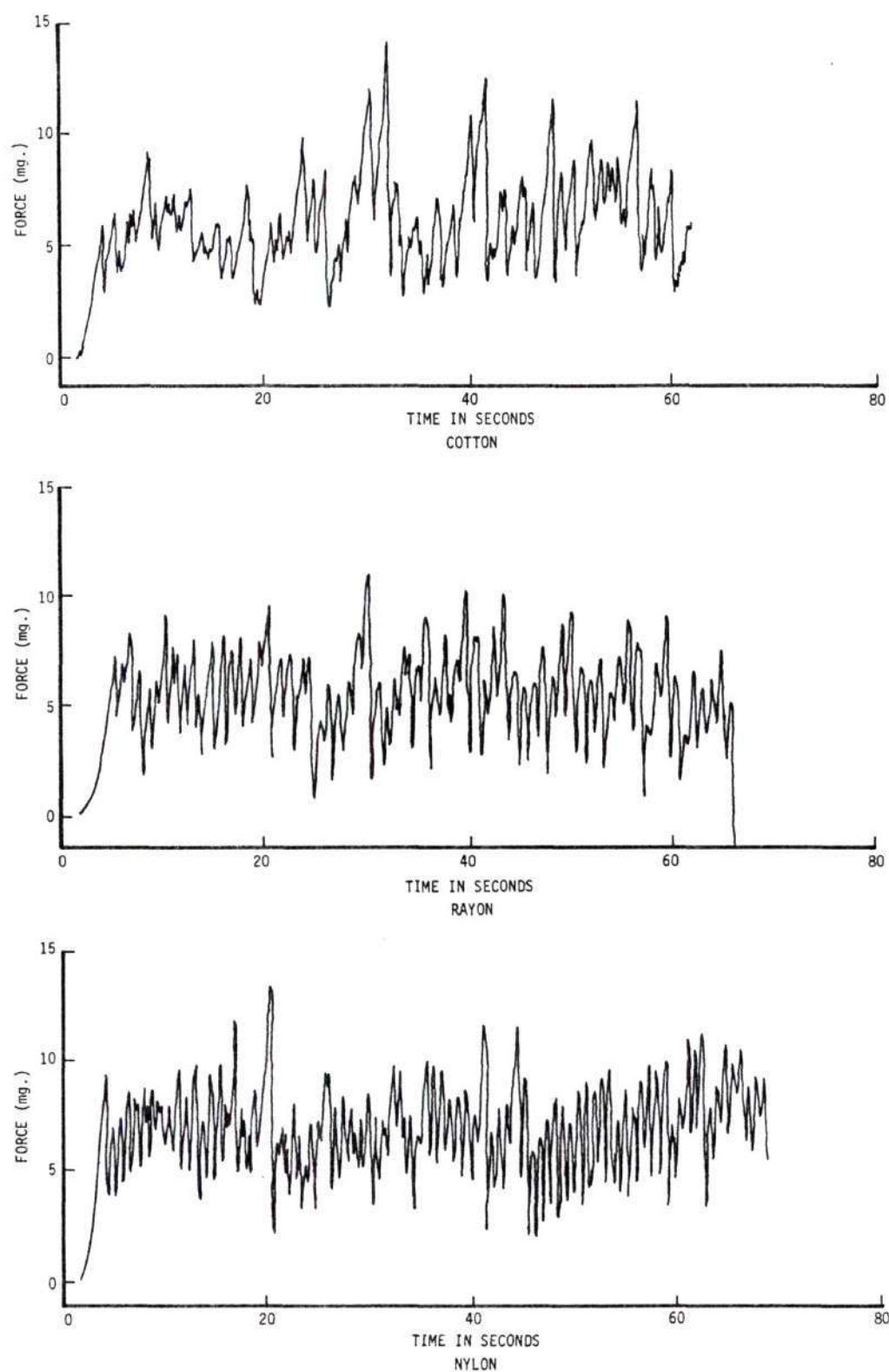


Figure 19. Typical Frictional Traces of Cotton, Rayon, and Nylon Fibers Showing Character Exhibited by these Fibers.

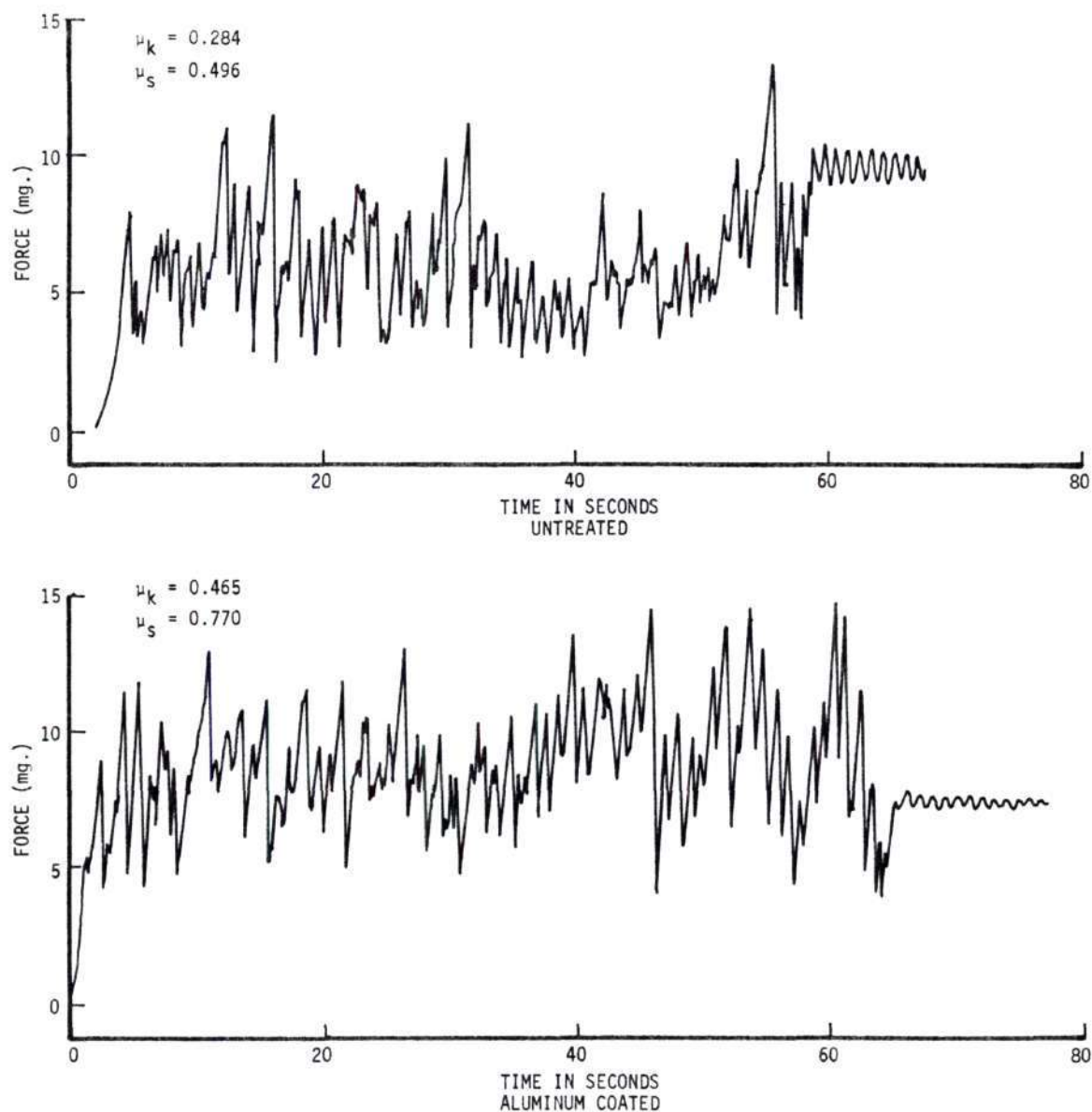


Figure 20. Typical Frictional Traces of Untreated and Aluminum Coated Cotton Fibers.

Table 1. Typical Values of the Coefficients of Friction for Various Textile Fibers

Investigator	Fiber	Method	Normal Force (mg)	Tension (mg)	μ_k	μ_s	μ_s/μ_k
Bryant	Cotton	Fiber/Fiber	20	425	0.292	0.523	1.8
Morrow	Cotton	Fiber/Pads			0.22		
Mercer and Makinson	Cotton	Fiber/Fiber	170-180			0.57	
Viswanathan	Cotton	Fringe/Fringe	30000			0.587	
Bryant	Viscose	Fiber/Fiber	20	425	0.304	0.539	1.8
Howell	Viscose	Fiber/Fiber	5.7	1000		0.460	
Mercer and Makinson	Viscose	Fiber/Fiber	170-180			0.190	
Gralen and Olofsson	Viscose	Fiber/Fiber	17		0.180	0.302	1.68
Gralen and Olofsson	Viscose	Fiber/Fiber	47		0.156	0.282	1.80
Gralen and Olofsson	Viscose	Fiber/Fiber	67		0.144	0.276	1.92
Guthrie and Oliver	Viscose	Fiber/Fiber	125		0.141	0.216	1.53
Viswanathan	Viscose	Fringe/Fringe	30000			0.546	
Bryant	Nylon	Fiber/Fiber	20	425	0.338	0.549	1.62
Howell	Nylon	Fiber/Fiber	15.4	1000		0.37	
Mercer and Makinson	Nylon	Fiber/Fiber	170-180			0.23	
Gralen and Olofsson	Nylon	Fiber/Fiber			0.400	0.470	1.18
Viswanathan	Nylon	Fringe/Fringe	30000			0.621	

Howell, using the apparatus shown in Figure 6 obtained a value of 0.37 for nylon at a normal force of 15.4 milligrams.²⁸ It was not specifically stated by Howell, but evidently this is the static frictional coefficient. He also reports a number of 0.46 for viscose, but this was for a normal load of only 5.7 milligrams.

Mercer and Makinson (Figure 3) have presented data on static coefficients of friction of various fibers at the high load range of 170 to 180 milligrams.²⁹ They report a value of 0.23 for 27 denier nylon, 0.19 for viscose rayon, and 0.57 for cotton. Even though the values for nylon and viscose are quite low (probably due to high normal force) the value of 0.57 for cotton is comparable to that of 0.523 obtained in this experiment on Empire WR using a tension of 425 milligrams and a normal force of 20 milligrams.

CHAPTER IV

DISCUSSION

Effect of Dynamics of System

It is obvious that if two fibers are not in constant contact with each other, that the frictional forces between them will vary and that the integrated value will be less than it would be if the normal force remained constant. Hence, an important need in tracking is for the arm when displaced by an asperity on the fiber surface to return rapidly to the correct normal force. If this time is significantly large, the assumption that the normal force is constant may introduce a major error into the calculations for the coefficient of friction.

A study involving the dynamics of the system employed showed that tracking was of prime importance. In an experiment in which balance arms of different moments of inertia were used, and hence of different oscillation periods, it was shown that the friction between two cotton fibers varied according to the moment of inertia of the arm used, even though the normal force established was the same for each arm.

Figure 11 depicts typical curves for 1-1/4 inch cotton fibers made at a normal load of approximately 20 milligrams for each of the three lever arms. The values of μ_k are 0.233, 0.212, and 0.172, respectively, for the Monel rod, the 3/16 inch aluminum rod and the 1/8 inch aluminum rod.

The moment of inertia of the lever arms was calculated from the

expression used to compute the period of a compound pendulum:

$$T = 2\pi \sqrt{\frac{I}{mgl}} \quad (\text{for small amplitudes})$$

where, T = the period, I = the moment of inertia, m = the mass of the arm, g = the acceleration of gravity, and l = the distance from the center of gravity to the point of suspension.

Using this formula the moments of inertia about the pivot point of the arms were calculated to be approximately 12,000, 3,000 and 1,150 gm./cm.² for the Monel, 3/16 inch aluminum, and the 1/8 inch aluminum arm respectively.

The stereomicroscopic and slow motion movie observations emphasized the necessity of employing an arm capable of superior tracking. It was apparent that if an arm with a low moment of inertia was used on the present apparatus that continuous contact between the fibers would be questionable due to oscillations in the arm. The additional difficulty of consistent tracking in the "slip" or dynamic portion of the stick-slip cycle would further disqualify low inertia arms in the apparatus.

The dynamics of the response time of the apparatus are also important to accuracy. An investigation of the accuracy of the obtained stick-slip curves made at various response times appears to be of great importance. The literature reveals that responses to frictional forces are made by springs, wires and other methods for which the response time is not defined by the author. The response time affects the values of the kinetic forces as measured by a planimeter or a similar integration of stick-slip traces.

In the case of stick-slip traces obtained by XY plotters, the pen traverse speed becomes an important factor. The slip portion of the trace becomes vital since at high speeds it tends to form a diagonal line as opposed a decisive and vertical slip. This action causes an increase in the area under the trace and therefore gives false data if not accounted for. This matter was discussed by Scheier and Lyons in a recent paper.³⁰ However, the motion pictures made during this investigation showed there was little contact between the fibers during the slip phase, and the principal error is introduced by the rate of pen traverse with respect to the recorder pen's Y-axis speed. Hence, slow traversing times are desirable.

Effect of Normal Force

The investigation of the effect of varying the normal force on the coefficients of friction for textile fibers showed in general that μ decreased as the normal force increased and approached some limiting value at high normal forces. Valid measurements with the present apparatus were obtained only at normal forces greater than 20 milligrams.

Variation of the coefficient of friction as affected by the normal force was measured for both cotton and nylon fibers. At forces below 20 milligrams the behavior of these two fibers was observed to be different. As the load approached zero, the friction between the cotton fibers dropped to a lower value along a curve that was extrapolated to intercept the zero force axis at about 0.16 whereas for nylon the coefficient of friction dropped to a value of 0.10 at 5.7 milligrams and appeared to be going to drop to near zero at 2 or 3 milligrams.

This behavior at low normal forces is thought to be mostly due to the dynamics of the system as well as surface characteristics of the fibers themselves. At low normal forces the fiber to fiber contact could be disturbed by the traveling fiber when some asperities make contact. This would tend to introduce a periodic oscillation in the arm and would not measure a correct value of the sliding force. The data obtained below 20 milligrams normal force by this instrument remains somewhat questionable.

The results obtained in this experiment above 20 milligrams normal force by varying the normal force are generally confirmed by other investigators in the field. Mercer and Makinson³¹ observed that the coefficient of friction for wool fibers decreases as the load increases (Figure 4). This phenomenon was observed for normal forces in the range from 1-20 milligrams. They pointed out that their measurements also became less accurate at the lower normal forces. Bowden and Tabor³² reported similar decreases in the coefficients of friction as the normal force was increased for many fibers over a large range of normal forces.

Gralen and Oloffson³³ reported that the frictional coefficient is approximately linearly dependent on the inverse value of the normal pressure (load) between two fibers. The slope of the line is assumed to be due to adhesion forces which are proportional to the area of contact. Their work was done in the normal force range from 17 to 97 milligrams.

Viswanathan³⁴ using a method of determining friction between fringes of fibers has also found that the coefficient of friction decreases

as the load is increased. His results are based on a multitude of tests on 30 cotton varieties and 15 various man-made fibers. He reported values of μ_s from 0.582 to 0.678 at a load of 30 grams for cotton. At 298 grams the value of μ_s dropped to a value range of 0.275 to 0.330.

When Morrow³⁶ investigated the effect of pressure (load) on the frictional coefficient of cotton by withdrawal of a fiber from between two fiber pads, he also reported that the coefficient decreases with increasing pressure.

From the results of this research and the results obtained by investigators in the field of fiber friction, it is evident that the expression $\mu = F/N$ does not remain constant at low normal forces for any given material. Rather, μ decreases to some limiting value with an increasing N . This effect has been observed by many scientists for both natural and man-made fibers over a range of N , restricted by the breaking strength of single fibers.

A rather significant observation from an investigation of this parameter is that the rate of change of μ decreases at higher normal forces. This would account for the essentially constant μ in general friction applications which are at relatively high normal force over large areas. Of special interest and more pertinent to textile interests are the values at low normal forces. Very few works appear to have presented reliable data in this region. Only that of Bowden and Tabor³⁶ appears to have been extensive and this was limited to the determination of μ_s values only. Hence better instrumentation and more significant measurements at low normal forces are especially necessary to future

comprehension of the role that friction plays in fiber processing.

Effect of Tension

A third parameter of some interest in fiber friction measurements is the tensile force used when mounting each fiber. A search of the literature reveals that very few investigators have studied the effect of the tensile force, or tension, using a fiber to fiber method of measurement. Most, if tension is mentioned at all, merely state that the tension was kept constant.

Measurements of the variation of the coefficients of friction obtained in this investigation on cotton fibers, as reported in Chapter III, by using tensile forces of 125, 425, 825, and 1,150 milligrams when mounting both fibers (Figure 14), indicated a reduction in μ_s and μ_k as the tension was increased. From the figure it can be seen that μ_k decreases from 0.356 at 125 milligrams tension to 0.236 at 1,150 milligrams tension. Similarly, μ_s decreases from 0.647 to 0.483 at these same tensions. The analyses of variance for both μ_k and μ_s as shown in Tables 7 and 8 indicate that the tension, or force under which the fibers were mounted, produces a statistically significant difference between these values at a 95 per cent level.

Observations indicated that the profile of the cotton fiber undergoes a change as a load is applied to one end. The first noticeable change is the disappearance of crimp which is eliminated with a few milligrams load for most practical purposes. Convolutions present a different problem since they cannot be eliminated and present a topographic surface absent in more regularly shaped fibers. This ready disappearance

of crimp and the continued existence of convolutions is discernible in Figure 15, previously exhibited. As the applied tension is increased from 125 milligrams to 1,150, the crimp is observed to disappear. The appearance of the convolutions change very little at the magnification utilized (136X). No literature was found describing the behavior of the convolutions under tension, except with respect to frequent occurrence of breakage at these points.

Comparison of these data with those obtained by others are reported below. Guthrie and Oliver³⁷ have reported an investigation of the effect of applied tension on viscose rayon. Their apparatus and results are shown in Figures 1 and 2. However, their results show an increase of frictional force with an increasing tension. This is the opposite of the effect for cotton reported in this work.

Their curves do not show data below 375 milligrams tension, but they do make the statement that μ_s and μ_k were observed to increase in this region, and pose the theory that this is due to the increase in the tangential force as given by the equation for friction between two cylindrical rods:

$$F_t = T_2 - T_1 = T_1(e^{\mu\theta} - 1)$$

where, F_t = tangential force, T_2 = leaving tension, T_1 = incoming tension, μ = coefficient of friction, and θ = angle of contact. The angle θ increases at low tensions.

Guthrie and Oliver also point out that the tension in the fibers depends partly on the normal load and that the true tension is therefore greater than the applied tension.

Hood³⁸ using a fiber twist method on natural and man-made fibers obtained data which correlated well with that found and reported in the present investigation on tension effects. His results are shown in Figure 5. The significance of this data is that it discounts the theory that man-made fibers might behave differently under tension than would natural fibers as related to frictional measurements.

Although neither of the authors cited presented theories as to why the coefficients of friction between single fibers are affected with an increase in tension, one explanation for this behavior is that tension affects cotton fibers in such a way as to cause the surface to become rigid, preventing the fibers from flexing around each other and decreasing the contact area. Flexing of fibers as discussed in the section dealing with stereomicroscopic observation of the stick-slip process, occurs in both the vertical and rotational directions. This may be described as rotation and "bowing" of the fibers. Both of these actions are reduced by the applied tension. Hence, the friction between the fibers is reduced.

Effect of Heat Cycling

A search of the literature revealed that no previous investigators had reported data on the effect of heat cycling on cotton fibers as related to their frictional properties. Morrow³⁹ did test cotton yarns in a temperature controlled enclosed box with the result that the friction decreased with increasing temperature. The only theory he offered for the observed behavior was that the absence of moisture in the yarns decreased the cohesive forces.

By heating cotton fibers in an oven and allowing them to recondition, effects of gin drying were simulated. Changes in the coefficients of friction as a result of this action indicated changes in the fiber surface condition.

The application of heat to cotton fibers produces several effects. The moisture content of the fibers is slow to recover and may not recover completely due to a hysteresis effect. Likewise, the fibers may lose a certain amount of their tensile strength at elevated temperatures which would be unrecoverable. In addition, the fibers may become embrittled at high temperatures due to breakage of chemical bonds in the cellulose structure.

It was not possible to obtain a visual determination of what happened to the natural waxes on the fibers, but it was suspected that these were adversely effected and would result in a more brittle fiber and subsequently yield a fiber with a higher frictional value. A. M. Goldfarb⁴⁰ has reported significant changes in the character of the convolutions of Empire WR at temperatures above 130 degrees C. The present investigation utilizes temperatures higher and lower than this value.

The results obtained confirmed a trend of the kinetic coefficient of friction to increase as the temperatures increased over the range from 70° C. to 220°C. Values for this coefficient of 0.285 and 0.327 were obtained for fibers treated at 70°C. and 220°C. respectively. Likewise, the static coefficient of friction increased from 0.590 to 0.629 over the same interval.

An analysis of variance of these data indicated that the differences

discerned are significant at the 90 per cent level. The variance calculations are shown in Tables 14 and 15 in the Appendix. In spite of this fact, the definite trend of increasing friction displayed in Figure 18 is believed to be valid and additional measurements would probably furnish sufficient data to establish a statistically significant finding at a higher confidence level. Duncan multiple range tests indicated the only significant differences at the 95 percent level was between the two extremes examined.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

The measured value of the frictional force between two textile fibers, when drawn one across the other, depends on variables in both the instrument design and mounting of the fibers.

The variables of importance in the instrument are resettable normal force, small displacement of the fixed fiber from the zero position, and its rapid restoration to the zero position by the selected restoring method. The instrument used in this investigation fulfilled these requirements with the exception that inaccuracies developed at normal forces below 20 milligrams.

The variables of importance with respect to the fiber are the force with which the fibers are mounted, the normal force between the fibers, and the alignment of the fibers with respect to each other and their respective driving or recording assemblies.

The kinetic coefficient of friction was observed to decrease by approximately 35 percent as the fiber mounting force or tension was increased from 125 milligrams to 1150 milligrams, and the static value decreased by approximately 25 percent at these same tensions. A normal force of 20 milligrams was adopted as a standard for this particular experiment. Indications are that the decrease occurred at a successively smaller rate at larger loads.

The coefficients of frictions between cotton fibers and between nylon fibers decreased, respectively, as the normal force between them was increased from 20 to 40 milligrams. The coefficients appeared to become asymptotic to a limiting minimum value as the normal force continued to increase.

The temperature cycling of cotton fibers to successive temperature plateaus between 70° and 220° C., similar to those that might be experienced in gin drying, resulted in a small increase in the coefficients of friction of the fibers. The coefficient of kinetic friction increased from 0.285 at 70°C. to 0.327 at 220°C. The static coefficient of friction increased from 0.590 to 0.629 at these same temperatures. This increase was small compared to the probable error of the experiment.

The character of the frictional data plots was dependent on the variety of fiber, and the ratio μ_s/μ_k was essentially a constant for a given fiber. The values for cotton ranged from 1.8 to 2.1 and appeared to be related to the surface and shape of the cotton. In contrast, a value of approximately 1.6 was found for smooth, cylindrical nylon. Rayon gave a μ_s/μ_k value of approximately 1.8 which approached the minimum ratio found for cotton. With these constants one can estimate μ_k values for measurements of other investigators presenting only μ_s values.

Recommendations

The existing apparatus needs to be further refined to investigate fiber friction on a more quantitative basis. The results in this investigation proved that it was capable of performing experiments when results

could be expressed in a semi-quantitative basis.

A major improvement would be the development of a system whereby the normal force between the fibers could be applied by an electromagnet. This would have the effect of damping the lever arm, thus reducing oscillation in the system caused by stick-slips or other factors. The device would enable the establishment of a known controlled normal force between the fibers at all times.

To reduce the time required for testing, it would be helpful to devise a better method of mounting the fiber on the galvanometer needle. Either a new holder design or a modification of the needle will be necessary to accomplish this objective. In addition to being quicker, an improved design would enable more precise alignment of the fibers. It was observed during this research that if the fiber on the galvanometer needle deviated from a parallel position, the base line inscribed by the XY plotter would change enough to give false results.

A new fiber holder arrangement would also permit testing of fibers at some angle other than 90° with respect to each other.

The method of calculating the static coefficient of friction should be changed. All maxima should be measured to obtain this average value.

The capability of measuring the frictional forces existing at very low normal forces (card webs) and the further investigation of the effects of fiber shapes on the character and resultant coefficients of friction is necessary to a proper understanding of the role of friction in textile processing.

APPENDIX

Table 2. Computed Friction Coefficients for Empire WR
With 125 Milligrams Tension in Each Fiber

Fiber Number	Coefficient of Kinetic Friction (μ_k)			Coefficient of Static Friction (μ_s)		
	a	b	c	a	b	c
1	.366	.321	.318	.600	.526	.533
2	.325	.375	.368	.600	.595	.711
3	.462	.336	.333	.785	.718	.715
4	.377	.312	.284	.737	.642	.615
5	.326	.384	.381	.695	.675	.732
6	.373	.346	.366	.627	.661	.628
7	.292	.360	.343	.717	.655	.736
8	.365	.309	.273	.508	.521	.503
9	.358	.368	.379	.592	.664	.618
10	.348	.384	.394	.620	.617	.622
11	.311	.367	.315	.561	.616	.562
12	.405	.320	.340	.783	.677	.615
13	.338	.332	.332	.639	.576	.562
14	.413	.354	.338	.635	.695	.633
15	.423	.445	.438	.783	.810	.800
Average	.365	.354	.347	.659	.643	.639
Grand Average		.356			.647	

Normal Force = 20 ± 1 mg.

Table 3. Computed Friction Coefficients for Empire WR with
425 Milligrams Tension in Each Fiber

Fiber Number	Coefficient of Kinetic Friction (μ_k)			Coefficient of Static Friction (μ_s)		
	a	b	c	a	b	c
1	.278	.248	.291	.523	.465	.474
2	.253	.252	.241	.453	.394	.442
3	.182	.251	.206	.413	.463	.415
4	.262	.270	.264	.493	.443	.458
5	.303	.321	.258	.546	.578	.485
6	.314	.258	.257	.513	.466	.508
7	.450	.505	.390	.781	.837	.581
8	.421	.348	.352	.639	.615	.677
9	.402	.324	.308	.677	.628	.547
10	.355	.333	.333	.620	.571	.594
11	.268	.257	.266	.522	.514	.547
12	.284	.261	.258	.496	.478	.503
13	.310	.294	.256	.529	.485	.475
14	.210	.197	.222	.429	.398	.400
15	.322	.252	.251	.528	.463	.452
Average	.308	.291	.277	.544	.520	.504
Grand Average		.292			.523	

Normal Force = 20 ± 1 mg.

Table 4. Computed Friction Coefficients for Empire WR
With 825 Milligrams Tension in Each Fiber

Fiber Number	Coefficient of Kinetic Friction (μ_k)			Coefficient of Static Friction (μ_s)		
	a	b	c	a	b	c
1	.443	.360	.376	.780	.684	.680
2	.264	.237	.245	.483	.473	.496
3	.450	.424	.373	.835	.795	.734
4	.210	.218	.224	.450	.496	.508
5	.309	.318	.280	.693	.670	.573
6	.191	.144	.197	.476	.451	.538
7	.316	.272	.282	.570	.540	.646
8	.226	.271	.278	.495	.548	.597
9	.310	.290	.211	.590	.630	.502
10	.288	.282	.268	.660	.645	.645
11	.247	.216	.339	.467	.419	.474
12	.291	.368	.248	.530	.649	.538
13	.236	.228	.223	.444	.426	.435
14	.208	.177	.164	.472	.444	.423
15	.185	.180	.131	.478	.468	.407
16	.184	.280	.278	.417	.473	.465
17	.272	.272	.288	.640	.626	.648
18	.272	.246	.224	.575	.534	.492
<hr/>						
Average	.272	.266	.257	.559	.554	.546
<hr/>						
Grand Average		.265			.552	

Normal Force = 20 ± 1 mg.

Table 5. Computed Friction Coefficients for Empire WR
With 1150 Milligrams Tension in Each Fiber

Fiber Number	Coefficient of Kinetic Friction (μ_k)			Coefficient of Static Friction (μ_s)		
	a	b	c	a	b	c
1	.244	.176	.218	.527	.470	.435
2	.279	.238	.312	.564	.493	.560
3	.185	.197	.198	.380	.375	.347
4	.197	.192	.166	.362	.354	.319
5	.196	.193	.192	.441	.411	.442
6	.196	.195	.237	.396	.350	.322
7	.368	.334	.362	.630	.498	.562
8	.286	.272	.260	.553	.490	.528
9	.289	.259	.291	.479	.478	.445
10	.192	.175	.211	.439	.396	.455
11	.260	.271	.251	.619	.607	.622
12	.303	.306	.240	.608	.641	.592
13	.254	.259	.229	.544	.578	.555
14	.244	.223	.180	.503	.507	.485
15	.168	.171	.167	.422	.454	.484
Average	.244	.231	.234	.498	.474	.477
Grand Average		.236			.483	

Normal Force = 20 ± 1 mg.

Table 6. Summary of Tension Data

Tension (mg.)	μ_k (Avg.)	μ_s (Avg.)	μ_s/μ_k (Avg.)
125	.356	.647	1.82
425	.292	.523	1.81
825	.265	.552	2.15
1150	.236	.483	2.08

Figure 7. Analysis of Variance for the Effect of Tension Upon the Kinetic Coefficient of Friction of Empire WR Cotton Fibers

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	Expected Mean Square	F Ratio
Tension	3	346226.95	115408.98	$\sigma_r^2 + 3\sigma_e^2 + 45 \sum_{i=1}^4 \frac{t_i^2}{3}$	17.13
Experimental Error	56	377184.71	6735.44	$\sigma_r^2 + 3\sigma_e^2$	
Measurement Error	120	262187.33	2184.89	σ_r^2	
Total	179	985598.99			

Conclusion: Tension treatments are significantly different at 99 percent level.

Figure 8. Analysis of Variance for the Effect of Tension upon the Static Coefficient of Friction of Empire WR Cotton Fibers

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	Expected Mean Square	F Ratio
Tension	3	661296.58	220432.19	$\sigma_r^2 + 3\sigma_e^2 + 45 \sum_{i=1}^4 \frac{t_i^2}{3}$	9.03
Experimental Error	56	1366941.96	24409.68	$\sigma_r^2 + 3\sigma_e^2$	
Measurement Error	120	212776.67	1773.14	σ_r^2	
Total	179	2241015.21			

Conclusion: Tension treatments are significantly different at 99 percent level.

Table 9. Computed Friction Coefficients for Empire WR
Subjected to 70°C. Temperature

Fiber Number	Coefficient of Kinetic Friction (μ_k)			Coefficient of Static Friction (μ_s)		
	A	B	C	A	B	C
1	.215	.221	.199	.573	.554	.525
2	.325	.278	.354	.670	.580	.615
3	.318	.352	.237	.562	.597	.495
4	.278	.290	.298	.580	.555	.598
5	.268	.291	.299	.508	.590	.568
6	.350	.309	.286	.755	.705	.638
7	.200	.212	.184	.421	.428	.425
8	.328	.305	.312	.730	.645	.720
9	.350	.340	.308	.658	.620	.615
Average	.292	.289	.275	.606	.586	.578
Grand Average	.285			.590		

Normal Force = 20 \pm 1 mg.

Tension = 425 mg.

Table 10. Computed Friction Coefficients for Empire WR
Subjected to 120°C Temperature

Fiber Number	Coefficient of Kinetic Friction (μ_k)			Coefficient of Static Friction (μ_s)		
	A	B	C	A	B	C
1	.330	.340	.339	.681	.708	.675
2	.342	.296	.292	.697	.603	.698
3	.306	.287	.247	.594	.590	.545
4	.258	.212	.246	.565	.444	.535
5	.321	.338	.272	.547	.617	.556
6	.281	.318	.250	.611	.682	.530
7	.261	.242	.244	.578	.547	.465
8	.306	.351	.310	.625	.673	.544
9	.299	.314	.301	.525	.650	.575
Average	.300	.300	.278	.603	.613	.569
Grand Average		.293			.595	

Normal Force = 20 \pm 1 mg.

Tension = 425 mg.

Table 11. Computed Friction Coefficients for Empire WR
Subjected to 170°C. Temperature

Fiber Number	Coefficient of Kinetic Friction (μ_k)			Coefficient of Static Friction (μ_s)		
	A	B	C	A	B	C
1	.292	.294	.286	.520	.530	.517
2	.386	.431	.386	.868	.875	.835
3	.312	.298	.319	.593	.600	.713
4	.276	.318	.310	.585	.646	.587
5	.309	.333	.305	.635	.659	.590
6	.292	.313	.355	.640	.700	.690
7	.353	.292	.322	.791	.668	.708
8	.316	.312	.315	.640	.642	.643
9	.296	.302	.254	.629	.557	.428
Average	.315	.321	.317	.656	.653	.635
Grand Average		.318			.648	

Normal Force = 20 ± 1 mg.
Tension = 425 mg.

Table 12. Computed Friction Coefficients for Empire WR
Subjected to 220°C Temperature

Fiber Number	Coefficient of Kinetic Friction (μ_k)			Coefficient of Static Friction (μ_s)		
	A	B	C	A	B	C
1	.376	.372	.355	.732	.645	.652
2	.337	.310	.355	.596	.586	.669
3	.404	.326	.339	.706	.610	.635
4	.312	.274	.322	.630	.588	.700
5	.373	.344	.337	.735	.699	.685
6	.264	.319	.256	.468	.545	.498
7	.391	.307	.318	.732	.635	.621
8	.306	.315	.356	.670	.638	.686
9	.295	.254	.304	.537	.471	.607
Average	.340	.313	.327	.645	.602	.639
Grand Average		.327			.629	

Normal Force = 20 ± 1 mg.
Tension = 425 mg.

Table 13. Summary of Heat Experiment

Temperature (°C)	μ_k (Avg.)	μ_s (Avg.)	μ_s/μ_k (Avg.)
70	.285	.590	2.09
120	.293	.595	2.04
170	.318	.648	2.03
220	.327	.629	1.93

Figure 14. Analysis of Variance for the Effect of Heat Upon the Kinetic Coefficient of Friction of Empire WR Cotton Fibers

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	Expected Mean Square	F Ratio
Heat	3	31415.26	10471.75	$\sigma_r^2 + 3\sigma_e^2 + 45 \sum_{i=1}^4 \frac{t_i^2}{3}$	2.53
Experimental Error	32	132353.93	4136.06	$\sigma_r^2 + 3\sigma_e^2$	
Measurement Error	72	49958.00	693.86	σ_r^2	
Total	107	213727.19			

Conclusions: 1. Treatments are significant at the 90 percent level.
 2. Duncan multiple range tests indicated only significant difference at 95 percent level was between 70°C treatment and 220°C treatment.

Figure 15. Analysis of Variance for the Effect of Heat Upon the Static Coefficient of Friction of Empire WR Cotton Fibers

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	Expected Mean Square	F Ratio
Heat	3	61905.21	20635.07	$\sigma_r^2 + 3\sigma_e^2 + 45 \sum_{i=1}^4 \frac{t_i^2}{3}$	1.10
Experimental Error	32	501422.44	18794.45	$\sigma_r^2 + 3\sigma_e^2$	
Measurement Error	72	169092.00	2348.50	σ_r^2	
Total	107	832419.65			

Conclusions: 1. Treatments are not significant at the 75 percent level.
 2. Duncan multiple range tests indicated no significant difference between treatments at 95 percent level.

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